7. Theory of Computation

Introduction to Theoretical CS

Two fundamental questions.

- What can a computer do?
- What can a computer do with limited resources?

General approach.

- Don't talk about specific machines or problems. (e.g., Pentium M running Linux kernel 2.6.15)
- Consider minimal abstract machines.
- Consider general classes of problems.

Why Learn Theory?

In theory ...

- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice ...

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

> In theory there is no difference between theory and practice. In practice there is. - Yogi Berra

Regular Expressions and DFAs

Pattern Matching Applications

Test if a string matches some pattern.

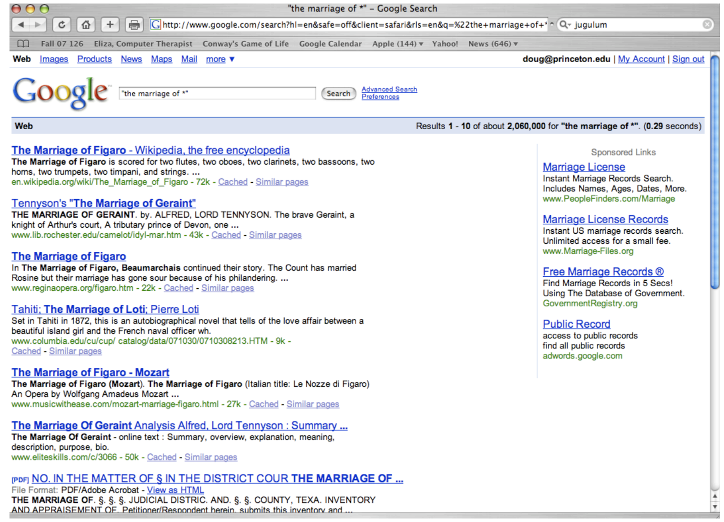
- Process natural language.
- Scan for virus signatures.
- Search for information using Google.
- Access information in digital libraries.
- Retrieve information from Lexis/Nexis.
- Search-and-replace in a word processors.
- Filter text (spam, NetNanny, ads, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).
- Search for markers in human genome using PROSITE patterns.

Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read in data stored in TOY input file format.
- Automatically create Java documentation from Javadoc comments.

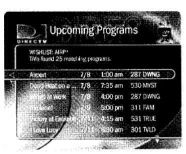
Pattern Matching in Google

Google. Supports * for full word wildcard and | for union.



Pattern Matching in TiVo

TiVo. WishList has very limited pattern matching.



Using * in WishList Searches. To search for similar words in Keyword and Title WishList searches, use the asterisk (*) as a special symbol that replaces the endings of words. For example, the keyword *AIRP** would find shows containing "airport," "airplane," "airplanes," as well as the movie "Airplane!" To enter an asterisk, press the SLOW ( ) button as you are spelling out your keyword or title.

The asterisk can be helpful when you're looking for a range of similar words, as in the example above, or if you're just not sure how something is spelled. Pop quiz: is it "irresistible" or "irresistable?" Use the keyword *IRRESIST** and don't worry about it! Two things to note about using the asterisk:

- It can only be used at a word's end; it cannot be used to omit letters at the beginning or in the middle of a word. (For example, *AIR*NE* or **PLANE* would not work.)

Reference: page 76, Hughes DirectTV TiVo manual

Describing a Pattern

PROSITE. Huge database of protein families and domains.

Q. How to describe a protein motif?

Ex. [signature of the C2H2-type zinc finger domain]

- C
- Between 2 and 4 amino acids.
- C
- 3 more amino acids.
- One of the following amino acids: LIVMFYWCX.
- 8 more amino acids.
- H
- Between 3 and 5 more amino acids.
- H

CAASCGGPYACGGWAGYHAGWH

Regular Expressions: Basic Operations

Regular expression. Notation to specify a set of strings.

Operation	Regular Expression	Yes	No
Concatenation	`aabaab`	aabaab	*every other string*
Wildcard	`.u.u.u.`	cumulus jugulum	succubus tumultuous
Union	`aa | baab`	aa baab	*every other string*
Closure	`ab*a`	aa abbba	ab ababa
Parentheses	`a(a|b)aab`	aaaab abaab	*every other string*
	`(ab)*a`	a ababababa	aa abbba

Regular Expressions: Examples

Regular expression. Notation is surprisingly expressive.

Regular Expression	Yes	No
`.*spb.*` *contains the trigraph* spb	raspberry crispbread	subspace subspecies
`a* | (a*ba*ba*ba*)*` *multiple of three* b's	bbb aaa bbbaababbaa	b bb baabbbaa
`.*0....` *fifth to last digit is* 0	1000234 98701234	111111111 403982772
`gcg(cgg|agg)*ctg` *fragile X syndrome indicator*	gcgctg gcgcggctg gcgcggaggctg	gcgcgg cggcggcggctg gcgcaggctg

Generalized Regular Expressions

Regular expressions are a standard programmer's tool.

- Built in to Java, Perl, Unix, Python, ….
- Additional operations typically added for convenience.
- Ex: `[a-e]+` is shorthand for `(a|b|c|d|e)(a|b|c|d|e)*`.

Operation	Regular Expression	Yes	No
One or more	`a(bc)+de`	abcde abcbcde	ade bcde
Character classes	`[A-Za-z][a-z]*`	lowercase Capitalized	camelCase 4illegal
Exactly k	`[0-9]{5}-[0-9]{4}`	08540-1321 19072-5541	111111111 166-54-1111
Negations	`[^aeiou]{6}`	rhythm	decade

Regular Expressions in Java

Validity checking. Is `input` in the set described by the `re`?

```
public class Validate {
   public static void main(String[] args) {
      String re    = args[0];
      String input = args[1];
      StdOut.println(input.matches(re));
   }
}
```

↑ powerful string library method

```
C2H2 type zinc finger domain
% java Validate "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H" CAASCGGPYACGGAAGYHAGAH
true
legal Java identifier
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true
valid email address (simplified)
% java Validate "[a-z]+@([a-z]+\.)+(edu|com)" doug@cs.princeton.edu
true
```

need quotes to "escape" the shell

More String Library Functions

String searching methods.

`public class String` *(Java's String library)*

`boolean`	`matches(String re)`	*does this string match the given regular expression*
`String`	`replaceAll(String re, String str)`	*replace all occurrences of regular expression with the replacement string*
`int`	`indexOf(String r, int from)`	*return the index of the first occurrence of the string r after the index from*
`String[]`	`split(String re)`	*split the string around matches of the given regular expression*

```
String s = StdIn.readAll();
s = s.replaceAll("\\s+", " ");
```

replace all sequences of whitespace characters with a single space

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`String[]`	`split(String re)`	*split the string around matches of the given regular expression*

```
String s = StdIn.readAll();
String[] words = s.split("\\s+");
```

create array of words in document

regular expression that matches any whitespace character

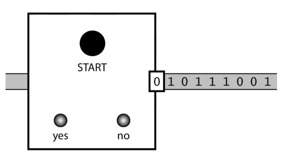
Solving the Pattern Match Problem

Regular expressions are a concise way to describe patterns.

- How would you implement the method `matches()` ?
- Hardware: build a deterministic finite state automaton (DFA).
- Software: simulate a DFA.

DFA: simple machine that solves a pattern match problem.

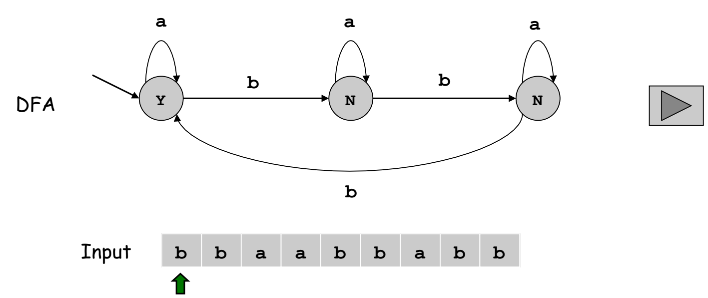
- Different machine for each pattern.
- Accepts or rejects string specified on input tape.
- Focus on `true` or `false` questions for simplicity.



Deterministic Finite State Automaton (DFA)

Simple machine with N states.

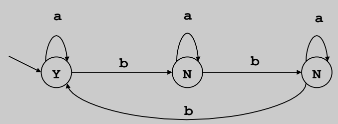
- Begin in start state.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- Accept input string if last state is labeled Y.



DFA and RE Duality

RE. Concise way to describe a set of strings.
DFA. Machine to recognize whether a given string is in a given set.

Duality. For any DFA, there exists a RE that describes the same set of strings; for any RE, there exists a DFA that recognizes the same set.



multiple of 3 b's

`a* | (a*ba*ba*ba*)*`

multiple of 3 b's

Practical consequence of duality proof: to match RE, (i) build DFA and (ii) simulate DFA on input string.

Implementing a Pattern Matcher

Problem. Given a RE, create program that tests whether given input is in set of strings described.

Step 1. Build the DFA.
- A compiler!
- See COS 226 or COS 320.

Step 2. Simulate it with given input.

```
State state = start;
while (!StdIn.isEmpty()) {
   char c = StdIn.readChar();
   state = state.next(c);
}
StdOut.println(state.accept());
```


Application: Harvester

Harvest information from input stream.

- Harvest patterns from DNA.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg
```

- Harvest email addresses from web for spam campaign.

```
% java Harvester "[a-z]+@([a-z]+\.)+(edu|com)" http://www.princeton.edu/~cos126
rs@cs.princeton.edu
dgabai@cs.princeton.edu
doug@cs.princeton.edu
wayne@cs.princeton.edu
```


Application: Harvester

Harvest information from input stream.
- Use `Pattern` data type to compile regular expression to NFA.
- Use `Matcher` data type to simulate NFA.

equivalent, but more efficient representation of a DFA

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester {
   public static void main(String[] args) {
      String re        = args[0];
      In in            = new In(args[1]);
      String input     = in.readAll();
      Pattern pattern  = Pattern.compile(re);
      Matcher matcher  = pattern.matcher(input);

      while (matcher.find()) {
         StdOut.println(matcher.group());
      }
   }
}
```

create NFA from RE

create NFA simulator

look for next match

the match most recently found

Application: Parsing a Data File

Ex: parsing an NCBI genome data file.

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus
ORIGIN
     1 tgtatttcat ttgaccgtgc tgttttttcc cggtttttca gtacggtgtt agggagccac
    61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
   121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
   ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

```
String re = "[ ]*[0-9]+([actg ]*).*";
Pattern pattern = Pattern.compile(re);
In in = new In(filename);
while (!in.isEmpty()) {
   String line = in.readLine();
   Matcher matcher = pattern.matcher(line);
   if (matcher.find()) {            // extract the RE part in parentheses
      String s = matcher.group(1).replaceAll(" ", "");
      // do something with s
   }
}
```


Limitations of DFA

No DFA can recognize the language of all bit strings with an equal number of 0's and 1's.

- Suppose an N-state DFA *can* recognize this language.
- Consider following input: 0000000011111111

  N+1 0's   N+1 1's
- DFA must accept this string.
- Some state **x** is revisited during first N+1 0's since only N states

  0000000011111111

  x   x

- Machine would accept same string without intervening 0's.

  0000011111111

  x
- This string doesn't have an equal number of 0's and 1's.

Summary

Programmer.
- Regular expressions are a powerful pattern matching tool.
- Implement regular expressions with finite state machines.

Theoretician.
- Regular expression is a compact description of a set of strings.
- DFA is an abstract machine that solves pattern match problem for regular expressions.
- DFAs and regular expressions have limitations.

Variations
- Yes (accept) and No (reject) states sometimes drawn differently
- Terminology: Deterministic Finite State Automaton (DFA), Finite State Machine (FSM), Finite State Automaton (FSA) are the same
- DFA's can have output, specified on the arcs or in the states
  - These may not have explicit Yes and No states

Fundamental Questions

Q. Are there patterns that cannot be described by any RE/DFA?

A. Yes.
- Bit strings with equal number of 0s and 1s.
- Decimal strings that represent prime numbers.
- DNA strings that are Watson-Crick complemented palindromes.
- and many, *many* more . . .

Q. Can we extend RE/DFA to describe richer patterns?

A. Yes.
- Context free grammar (e.g., Java).
- Turing machines.

Turing Machines

Challenge: Design simplest machine that is "as powerful" as conventional computers.

Alan Turing (1912-1954)

Turing Machine

Desiderata. Simple model of computation that is "as powerful" as conventional computers.

Intuition. Simulate how humans calculate.

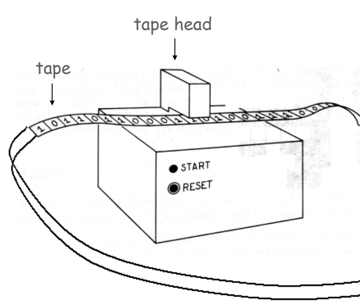
Ex. Addition.

			1	2	3	4	5	6		
		+	3	1	4	1	5	9		

Turing Machine: Tape

Tape.
- Stores input, output, and intermediate results.
- One arbitrarily long strip, divided into cells.
- Finite alphabet of symbols.

Tape head.
- Points to one cell of tape.
- Reads a symbol from active cell.
- Writes a symbol to active cell.
- Moves left or right one cell at a time.



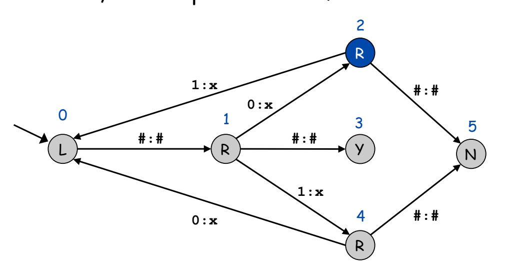
tape head ↓

tape	...	#	1	1	0	0	+	1	0	1	1	#	...

Turing Machine: Fetch, Execute

States.
- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

State transition diagram.
- Ex. if in state 2 and input symbol is 1 then: overwrite the 1 with x, move to state 0, move tape head to left.

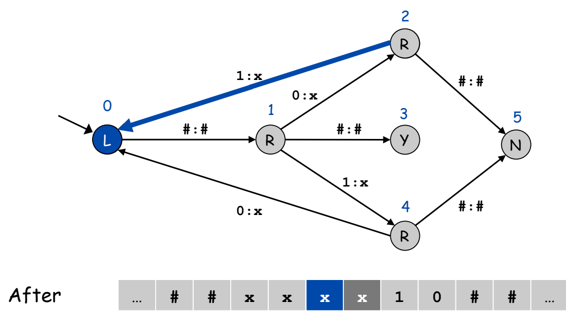


Before	...	#	#	x	x	x	1	1	0	#	#	...

Turing Machine: Fetch, Execute

States.
- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

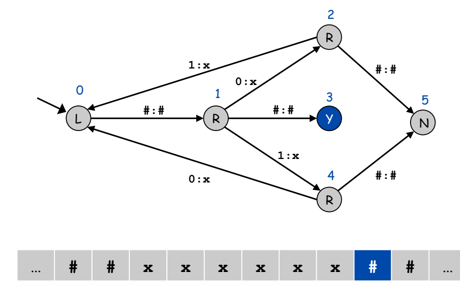
State transition diagram.
- Ex. if in state 2 and input symbol is `1` then: overwrite the `1` with `x`, move to state 0, move tape head to left.



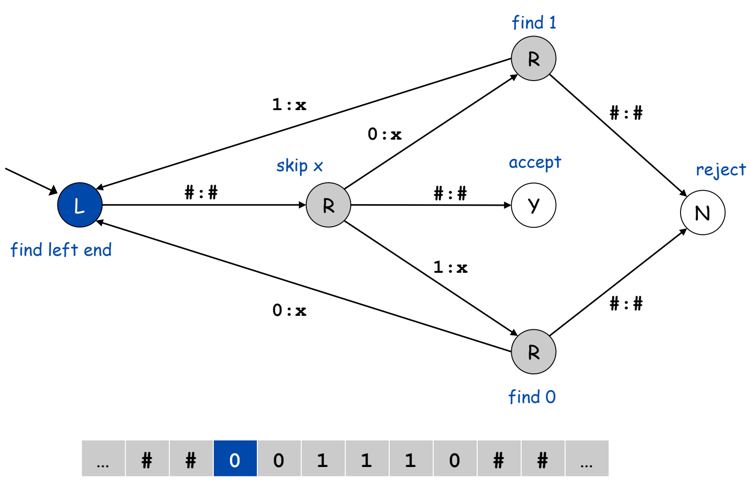
Turing Machine: Initialization and Termination

Initialization.
- Set input on some portion of tape.
- Set tape head.
- Set initial state.

Termination.
- Stop if enter `yes`, `no`, or `halt` state.
- Infinite loop possible.
  - (definitely stay tuned !)



Example: Equal Number of 0's and 1's



Turing Machine Summary

Goal: simplest machine that is "as powerful" as conventional computers.

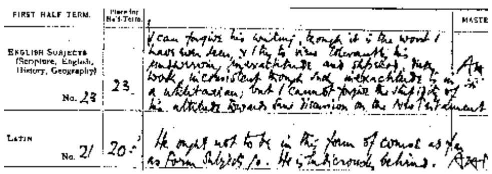
Surprising Fact 1. Such machines are very simple: TM is enough!
Surprising Fact 2. Some problems cannot be solved by ANY computer.
↑ next lecture

Consequences.
- Precursor to general purpose programmable machines.
- Exposes fundamental limitations of all computers.
- Enables us to study the physics and universality of computation.
- No need to seek more powerful machines!

Variations
- Instead of just recognizing strings, TM's can produce output: the contents of the tape
- Instead of Y and N states, TM's can have a plain Halt state

Alan Turing (1912-1954).

- Father of computer science.
- Computer Science's "Nobel Prize" is called the Turing Award.



FIRST HALF TERM.	Place for Half-Term.		MASTER
ENGLISH SUBJECTS (Scripture, English, History, Geography) No. 23	23	I can forgive his writing, though it is the worst I have ever seen, & I try to view tolerantly his unsatisfactory, [illegible] and slipshod, dirty work, inconsistent though such [illegible] is in a utilitarian; but I cannot forgive the stupidity of his attitude towards sane discussion on the New Testament	[illegible]
LATIN No. 21	20	He ought not to be in this form of course as far as form subjects go. He is [illegible] behind.	[illegible]

Alan's report card at 14.

Alan Turing and his elder brother.