COS 429: COMPUTER VISION
RADIOMETRY (1 lecture)

- Elements of Radiometry
- Radiance
- Irradiance
- BRDF
- Photometric Stereo

Reading: Chapters 4 and 5

Many of the slides in this lecture are courtesy to Prof. J. Ponce
Geometry  Viewing  Lighting  Surface albedo

Images I
Photometric stereo example

Image Formation: Radiometry

What determines the brightness of an image pixel?

- The light source(s)
- The surface normal
- The surface properties
- The optics
- The sensor characteristics
The Illumination and Viewing Hemi-sphere

At infinitesimal, each point has a tangent plane, and thus a hemisphere $\Omega$.

The ray of light is indexed by the polar coordinates $(\theta, \varphi)$. 

\[ \mathbf{n}(x, y) \]
Foreshortening

• **Principle:** two sources that look the same to a receiver must have the same effect on the receiver.

• **Principle:** two receivers that look the same to a source must receive the same amount of energy.

• “look the same” means produce the same input hemisphere (or output hemisphere)

• **Reason:** what else can a receiver know about a source but what appears on its input hemisphere? (ditto, swapping receiver and source)

• **Crucial consequence:** a big source (resp. receiver), viewed at a glancing angle, must produce (resp. experience) the same effect as a small source (resp. receiver) viewed frontally.
Measuring Angle

• To define radiance, we require the concept of solid angle
• **The solid angle subtended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P**
  • Measured in *steradians*, sr
  • Definition is analogous to projected angle in 2D
  • If I’m at P, and I look out, solid angle tells me how much of my view is filled with an object
Solid Angle of a Small Patch

- Later, it will be important to talk about the solid angle of a small piece of surface
DEFINITION: Angles and Solid Angles

\[ \theta \]

\[ \Omega \]

(radians)

(steradians)
DEFINITION: The radiance is the power traveling at some point in a given direction per unit area perpendicular to this direction, per unit solid angle.

\[ \delta^2 P = L(P, \nu) \delta A \delta \omega \]

\[ \delta^2 P = L(P, \nu) \cos \theta \delta A \delta \omega \]
PROPERTY: Radiance is constant along straight lines (in vacuum).

\[ \delta^2 P = L(P, v) \delta A \delta \omega \]

\[ \delta^2 P = L(P, v) \cos \theta \delta A \delta \omega \]
DEFINITION: Irradiance

The irradiance is the power per unit area incident on a surface.

\[ \delta^2 P = \delta E \delta A = L_i(P, v_i) \cos \theta_i \delta \omega_i \delta A \]

\[ \delta E = L_i(P, v_i) \cos \theta_i \delta \omega_i \]

\[ E = \int_H L_i(P, v_i) \cos \theta_i \, d\omega_i \]
Photometry

- $L$ is the radiance.
- $E$ is the irradiance.
DEFINITION: The Bidirectional Reflectance Distribution Function (BRDF)

$$L_o(P, v_o) = \rho_{BD}(P, v_i, v_o) \delta E_i(P, v_i)$$

$$= \rho_{BD}(P, v_i, v_o) L_i(P, v_i) \cos \theta_i \delta \omega_i$$

Helmoltz reciprocity law: $\rho_{BD}(P, v_i, v_o) = \rho_{BD}(P, v_o, v_i)$

The BRDF is the ratio of the radiance in the outgoing direction to the incident irradiance ($\text{sr}^{-1}$).
DEFINITION: Radiosity

The radiosity is the total power leaving a point on a surface per unit area (W * m⁻²).

\[ B(P) = \int_H L_o(P, \nu_o) \cos \theta_o \, d\omega \]

Important case: \( L_o \) is independent of \( \nu_o \).

\[ B(P) = \pi L_o(P) \]
DEFINITION: Lambertian (or Matte) Surfaces

A Lambertian surface is a surface whose BRDF is independent of the outgoing direction (and by reciprocity of the incoming direction as well).

\[ \rho_{BD}(v_i, v_o) = \rho_{BD} = \text{constant}. \]

The albedo is \( \rho_d = \pi \rho_{BD}. \)
DEFINITION: Specular Surfaces as Perfect or Rough Mirrors

Perfect mirror: \( L_o(P, \nu_s) = L_i(P, \nu_i) \)

Phong (non-physical model): \( L_o(P, \nu_o) = \rho_s L_i(P, \nu_i) \cos^n \delta \)

Hybrid model: \( L_o(P, \nu_o) = \rho_d \int H L_i(P, \nu_i) \cos \theta_i d\omega_i + \rho_s L_i(P, \nu_i) \cos^n \delta \)
A point light source is an idealization of an emitting sphere with radius $\varepsilon$ at distance $R$, with $\varepsilon << R$ and uniform radiance $L_e$ emitted in every direction.

For a Lambertian surface, the corresponding radiosity is
Local Shading Model

- Assume that the radiosity at a patch is the sum of the radiosities due to light source and sources alone.

\[ \sum \rho \cdot = \sum P B \cdot \text{visible} \ \text{(for point sources at infinity)} \]

No interreflections.

- For point sources:

- For point sources at infinity:
Photometric Stereo (Woodham, 1979)

Problem: Given $n$ images of an object, taken by a fixed camera under different (known) light sources, reconstruct the object shape.
Photometric Stereo: Example (1)

- Assume a Lambertian surface and distant point light sources.

\[ I(P) = kB(P) = k\rho \mathbf{N}(P) \cdot \mathbf{S} = g(P) \cdot \mathbf{V} \]

with \( g(P) = \rho \mathbf{N}(P) \) and \( \mathbf{V} = k \mathbf{S} \)

- Given \( n \) images, we obtain \( n \) linear equations in \( g \):

\[
\begin{align*}
I_1 \quad I_2 \quad \ldots \quad I_n \\
\mathbf{V}_1 \cdot \mathbf{g} \quad \mathbf{V}_2 \cdot \mathbf{g} \quad \ldots \quad \mathbf{V}_n \cdot \mathbf{g} \\
\mathbf{V}_1^T \quad \mathbf{V}_2^T \quad \ldots \quad \mathbf{V}_n^T
\end{align*}
\]

\[
\mathbf{g} \quad \mathbf{i} = \mathbf{V} \mathbf{g} \quad \mathbf{g} = \mathbf{V}^+ \mathbf{i}
\]
Photometric Stereo: Example (2)

- What about shadows?

- Just skip the equations corresponding to zero-intensity pixels.

- Only works when there is no ambient illumination.
Photometric Stereo: Example (3)

\[ g(P) = \rho(P)N(P) \]

\[ \rho(P) = |g(P)| \]
Photometric Stereo: Example (3)

\[ \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} \right) \]

Integrability!
Photometric stereo example