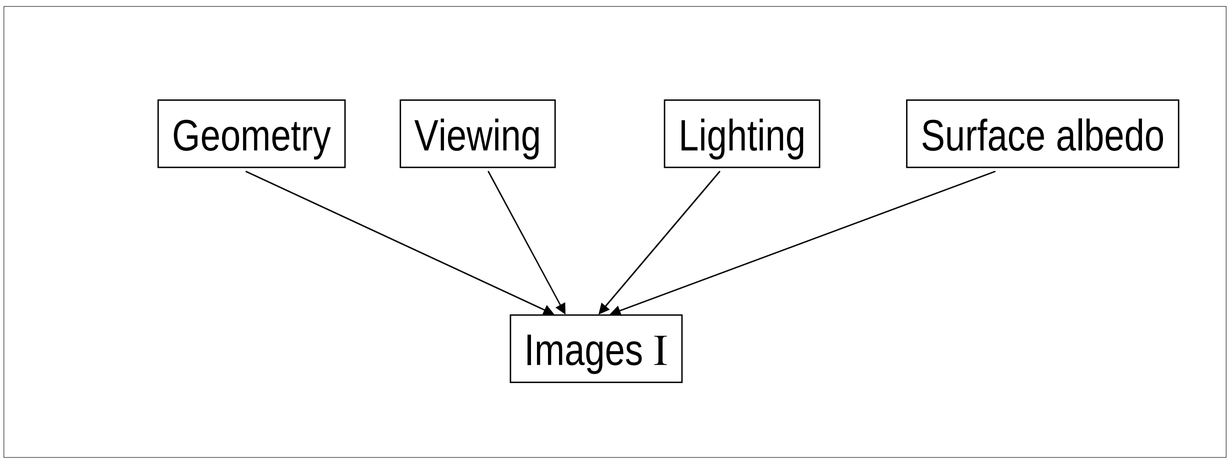
COS 429: COMPUTER VISON

RADIOMETRY (1 lecture)

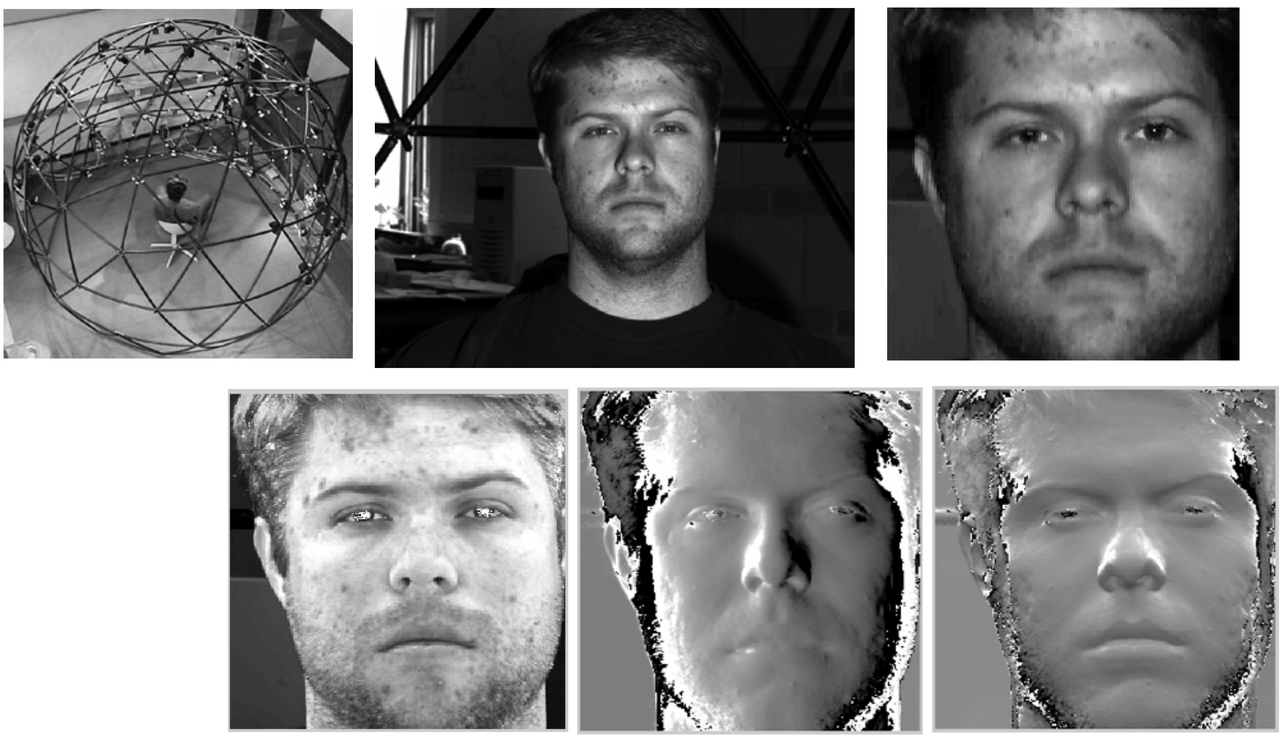
- Elements of Radiometry
- Radiance
- Irradiance
- BRDF
- Photometric Stereo

Reading: Chapters 4 and 5

Many of the slides in this lecture are courtesy to Prof. J. Ponce

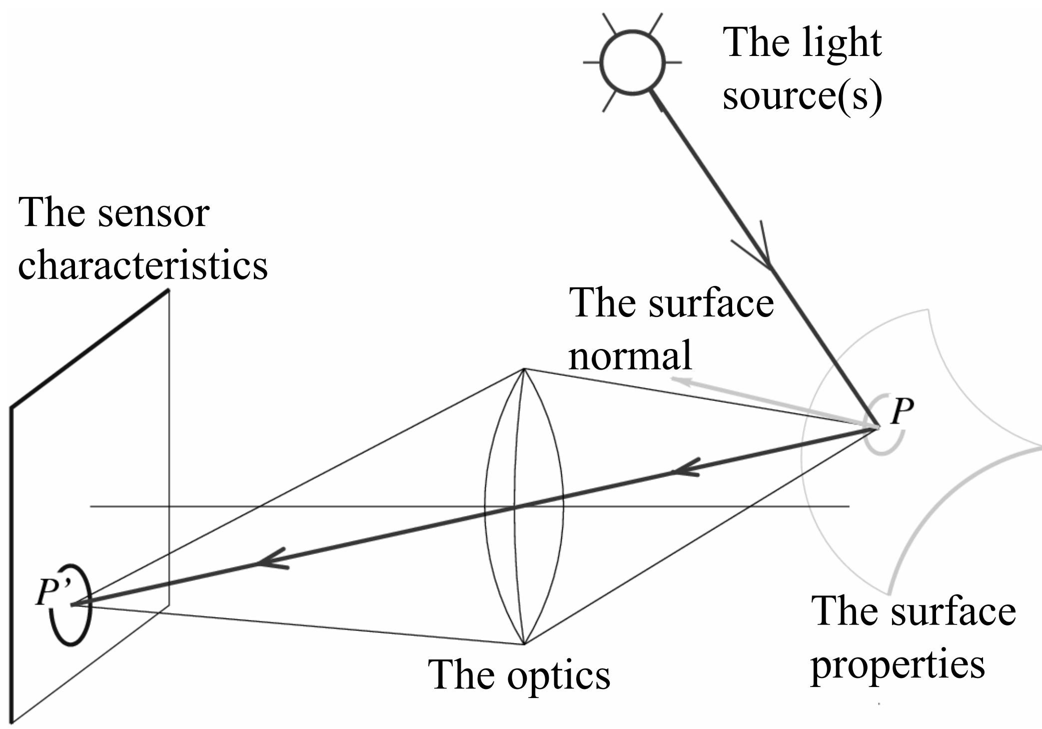

Geometry
Viewing
Lighting
Surface albedo
Images I

Photometric stereo example



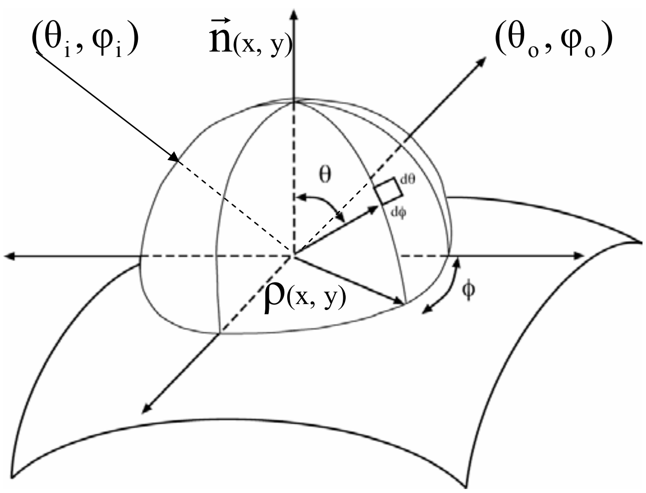
data from: http://www1.cs.columbia.edu/~belhumeur/pub/images/yalefacesB/readme

Image Formation: Radiometry



What determines the brightness of an image pixel?

The Illumination and Viewing Hemi-sphere

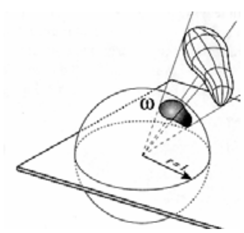


At infinitesimal, each point has a tangent plane, and thus a hemisphere Ω.

The ray of light is indexed by the polar coordinates

Foreshortening

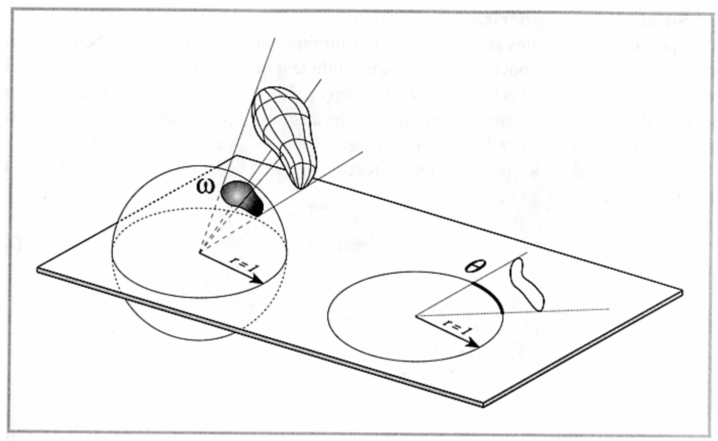
- **Principle:** two sources that look the same to a receiver must have the same effect on the receiver.
- **Principle:** two receivers that look the same to a source must receive the same amount of energy.
- "look the same" means produce the same input hemisphere (or output hemisphere)



- **Reason:** what else can a receiver know about a source but what appears on its input hemisphere? (ditto, swapping receiver and source)
- **Crucial consequence:** a big source (resp. receiver), viewed at a glancing angle, must produce (resp. experience) the same effect as a small source (resp. receiver) viewed frontally.

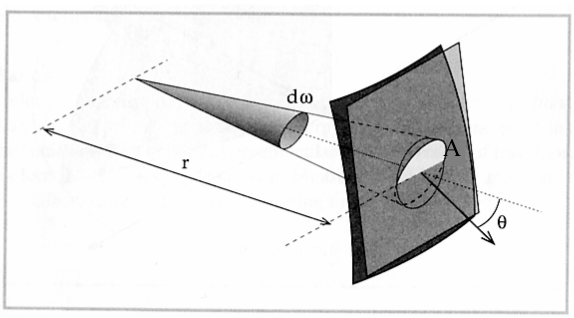
Measuring Angle

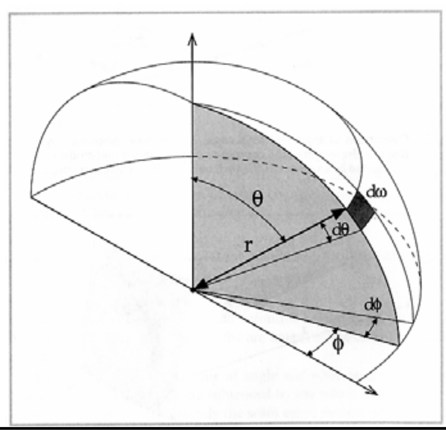
- To define radiance, we require the concept of solid angle
- **The solid angle sub-tended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P**
- Measured in *steradians*, sr
- Definition is analogous to projected angle in 2D
- If I'm at P, and I look out, solid angle tells me how much of my view is filled with an object



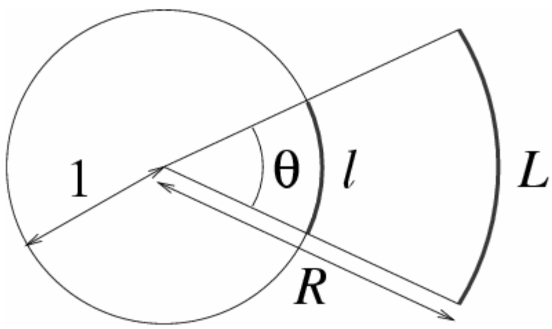
Solid Angle of a Small Patch

- Later, it will be important to talk about the solid angle of a small piece of surface

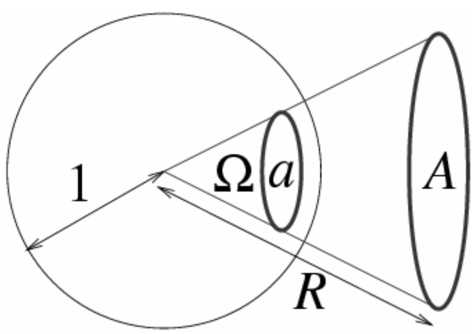




DEFINITION: Angles and Solid Angles

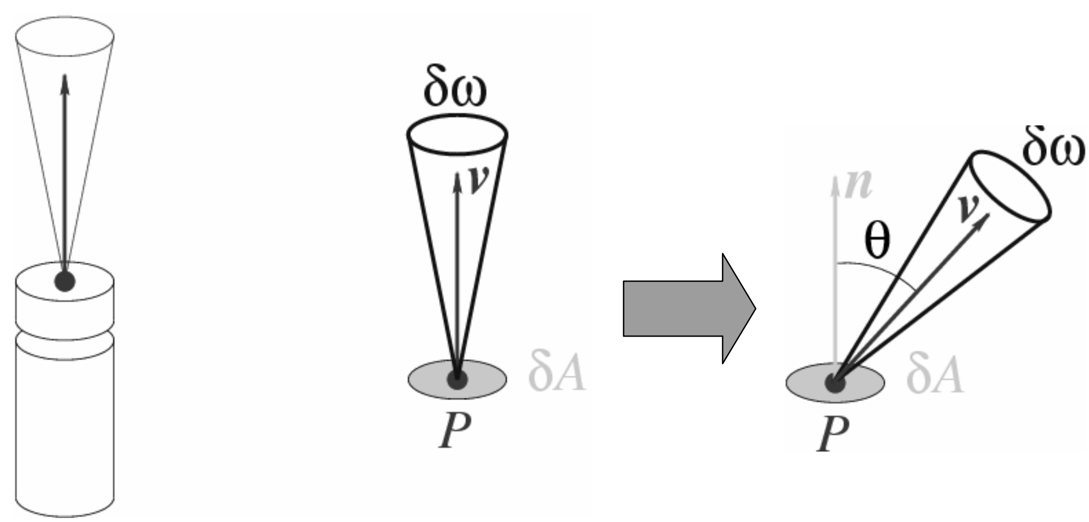


(radians)



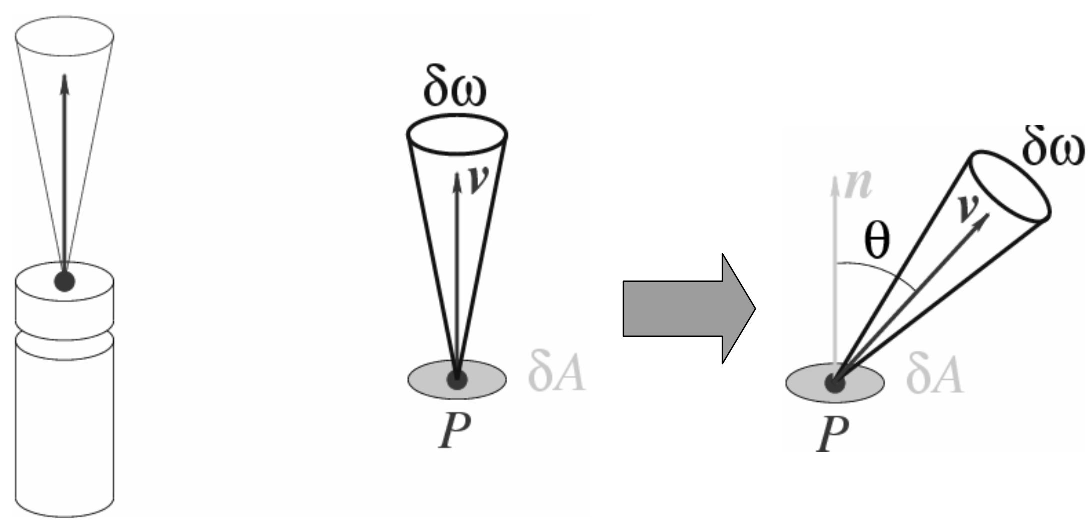
(steradians)

DEFINITION: The radiance is the power traveling at some point in a given direction per unit area perpendicular to this direction, per unit solid angle.



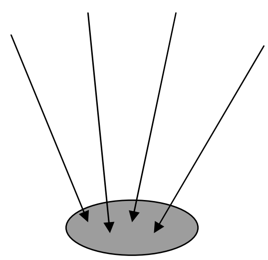
$\delta^2 P = L(P, \boldsymbol{v})\, \delta A\, \delta\omega$ → $\delta^2 P = L(P, \boldsymbol{v}) \cos\theta\, \delta A\, \delta\omega$

PROPERTY: Radiance is constant along straight lines (in vacuum).

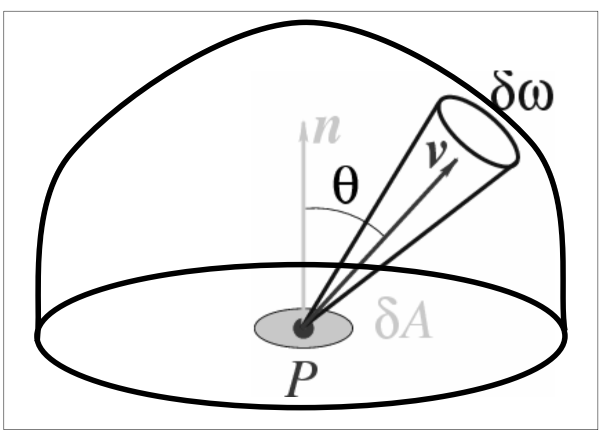


$\delta^2 P = L(P, \boldsymbol{v})\, \delta A\, \delta\omega$ ⇒ $\delta^2 P = L(P, \boldsymbol{v}) \cos\theta\, \delta A\, \delta\omega$

DEFINITION: Irradiance



The irradiance is the power per unit area incident on a surface.

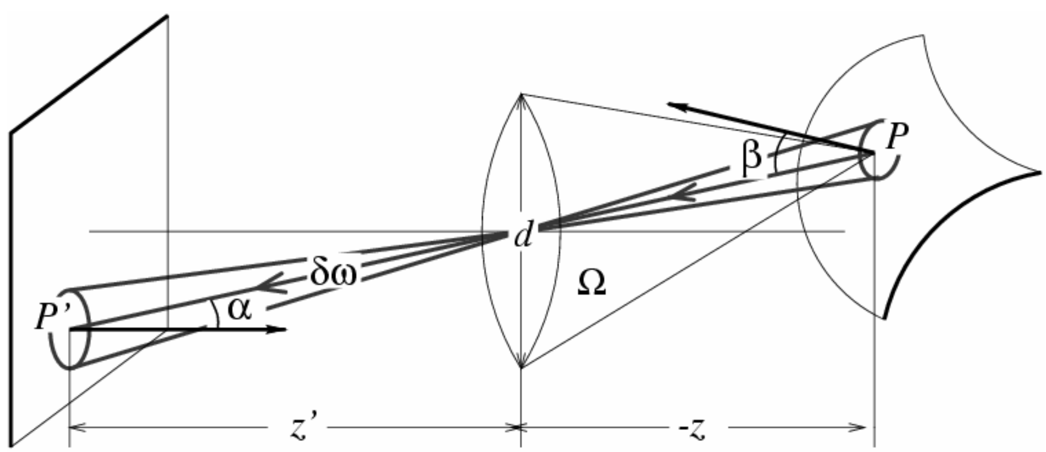


$$\delta^2 P = \delta E \, \delta A = L_i(P , \boldsymbol{v}_i) \cos\theta_i \delta\omega_i \, \delta A$$

$$\delta E = L_i(P , \boldsymbol{v}_i) \cos\theta_i \, \delta\omega_i$$

$$E = \int_H L_i (P, v_i) \cos\theta_i \, d\omega_i$$

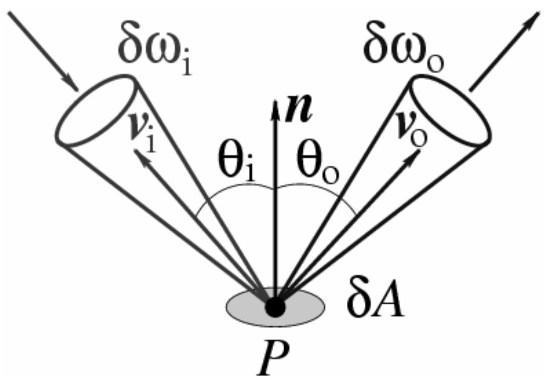
Photometry



• *L* is the radiance.

• *E* is the irradiance.

DEFINITION: The Bidirectional Reflectance Distribution Function (BRDF)



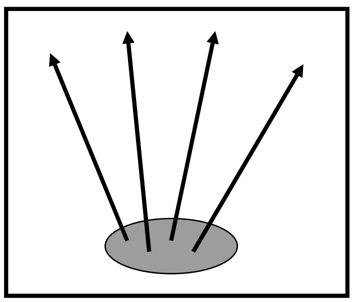
The BRDF is the ratio of the radiance in the outgoing direction to the incident irradiance (sr^{-1}).

$$L_o (P, \boldsymbol{v}_o) = \rho_{BD} (P, \boldsymbol{v}_i , \boldsymbol{v}_o) \, \delta E_i (P, \boldsymbol{v}_i)$$

$$= \rho_{BD} (P, \boldsymbol{v}_i , \boldsymbol{v}_o) \; L_i (P, \boldsymbol{v}_i) \cos \theta_i \; \delta\omega_i$$

Helmoltz reciprocity law: $\rho_{BD} (P, \boldsymbol{v}_i , \boldsymbol{v}_o) = \rho_{BD} (P, \boldsymbol{v}_o , \boldsymbol{v}_i)$

DEFINITION: Radiosity



The radiosity is the total power Leaving a point on a surface per unit area (W * m^{-2}).

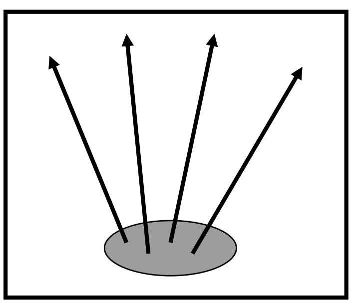
$$B(P) = \int_H L_o (P , \boldsymbol{v}_o) \cos\theta_o \, d\omega$$

Important case: L_o is independent of $\boldsymbol{v}_o$.

$$B(P) = \pi L_o(P)$$

DEFINITION: Lambertian (or Matte) Surfaces

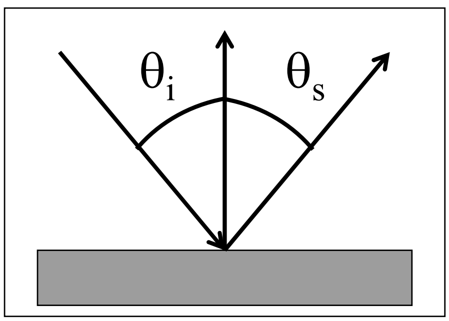
A Lambertian surface is a surface whose BRDF is independent of the outgoing direction (and by reciprocity of the incoming direction as well).



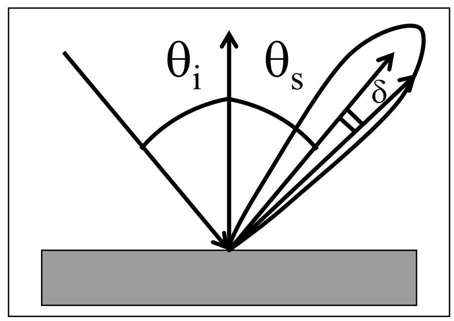
$$\rho_{BD}(\boldsymbol{v}_i , \boldsymbol{v}_o) = \rho_{BD} = \text{constant.}$$

The albedo is $\rho_d = \pi \, \rho_{BD}$.

DEFINITION: Specular Surfaces as Perfect or Rough Mirrors



Perfect mirror



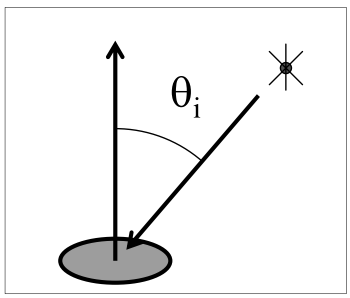
Rough mirror

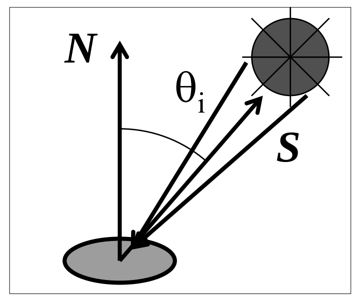
Perfect mirror: $L_o (P, \boldsymbol{v}_s) = L_i (P, \boldsymbol{v}_i)$

Phong (non-physical model): $L_o (P, \boldsymbol{v}_o) = \rho_s L_i(P, \boldsymbol{v}_i) \cos^n \delta$

Hybrid model: $L_o (P, \boldsymbol{v}_o) = \rho_d \int_H L_i(P, \boldsymbol{v}_i) \cos\theta_i \, d\omega_i + \rho_s L_i(P, \boldsymbol{v}_i) \cos^n \delta$

DEFINITION: Point Light Sources





A point light source is an idealization of an emitting sphere with radius ε at distance R, with ε << R and uniform radiance L_e emitted in every direction.

For a Lambertian surface, the corresponding radiosity is

Local Shading Model

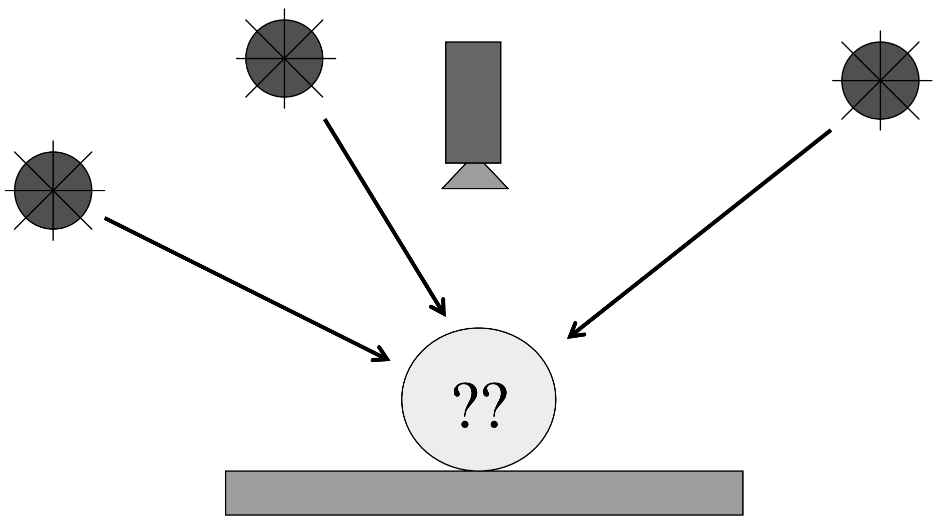
- Assume that the radiosity at a patch is the sum of the radiosities due to light source and sources alone.

  No interreflections.

- For point sources:

- For point sources at infinity:

Photometric Stereo (Woodham, 1979)



Problem: Given n images of an object, taken by a fixed camera under different (known) light sources, reconstruct the object shape.

Photometric Stereo: Example (1)

- Assume a Lambertian surface and distant point light sources.

$$I(P) = kB(P) = k\rho\,\mathbf{N}(P) \bullet \mathbf{S} = \mathbf{g}(P) \bullet \mathbf{V}$$

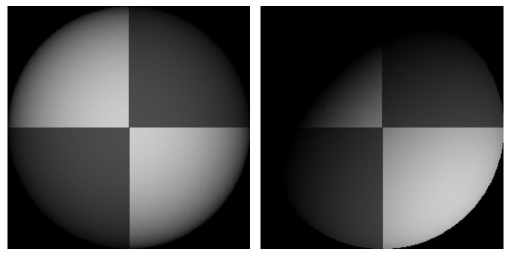
with $\mathbf{g}(P) = \rho\,\mathbf{N}(P)$ and $\mathbf{V} = k\,\mathbf{S}$

- Given n images, we obtain n linear equations in ***g***:

$$\mathbf{i} = \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \cdot \mathbf{g} \\ \mathbf{V}_2 \cdot \mathbf{g} \\ \dots \\ \mathbf{V}_n \cdot \mathbf{g} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \\ \mathbf{V}_n^T \end{bmatrix} \mathbf{g} \Rightarrow \mathbf{i} = \boldsymbol{V}\,\mathbf{g} \Rightarrow \boxed{\mathbf{g} = \mathcal{V}^{-1}\,\mathbf{i}}$$

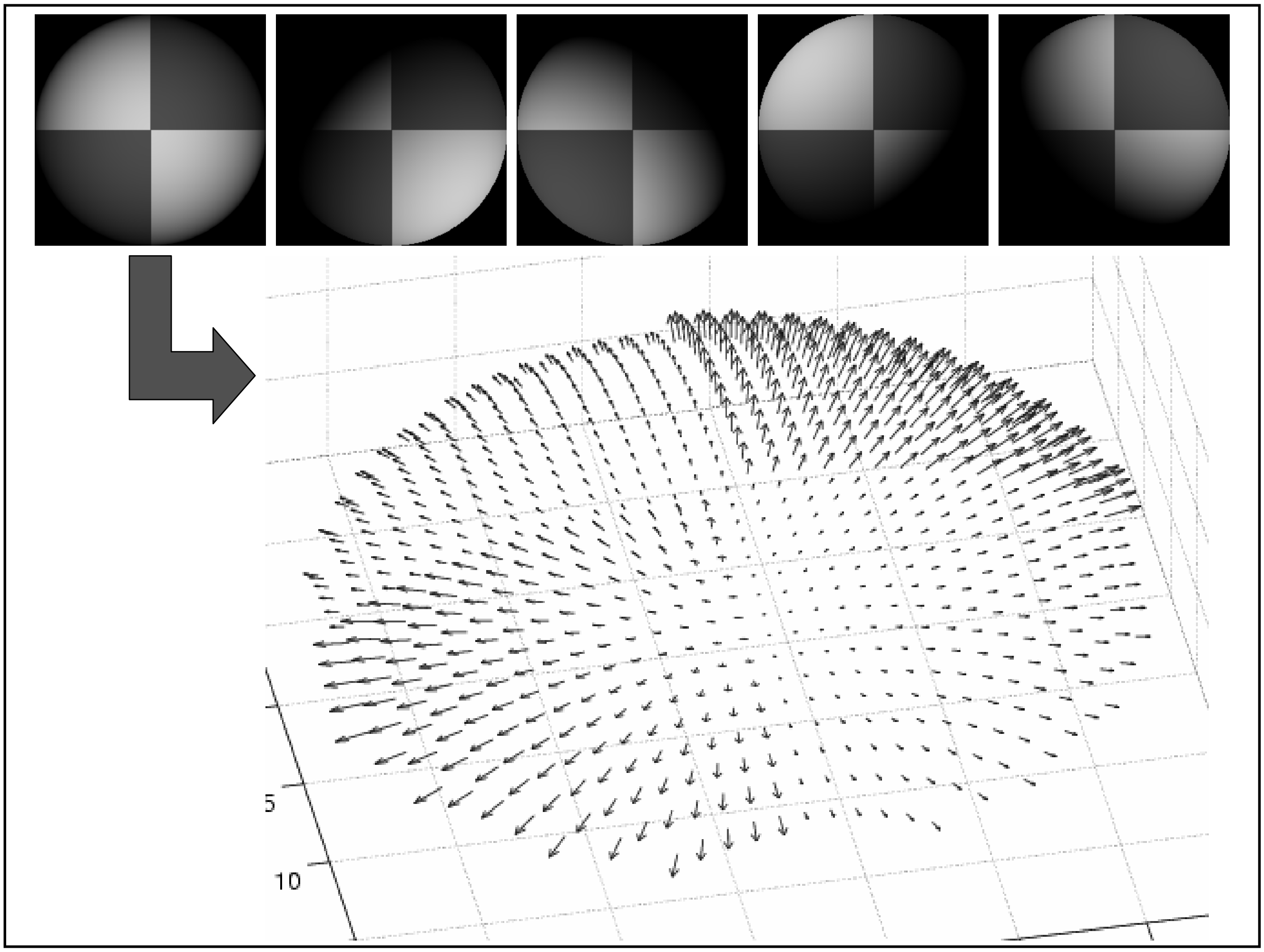
Photometric Stereo: Example (2)

- What about shadows?



- Just skip the equations corresponding to zero-intensity pixels.

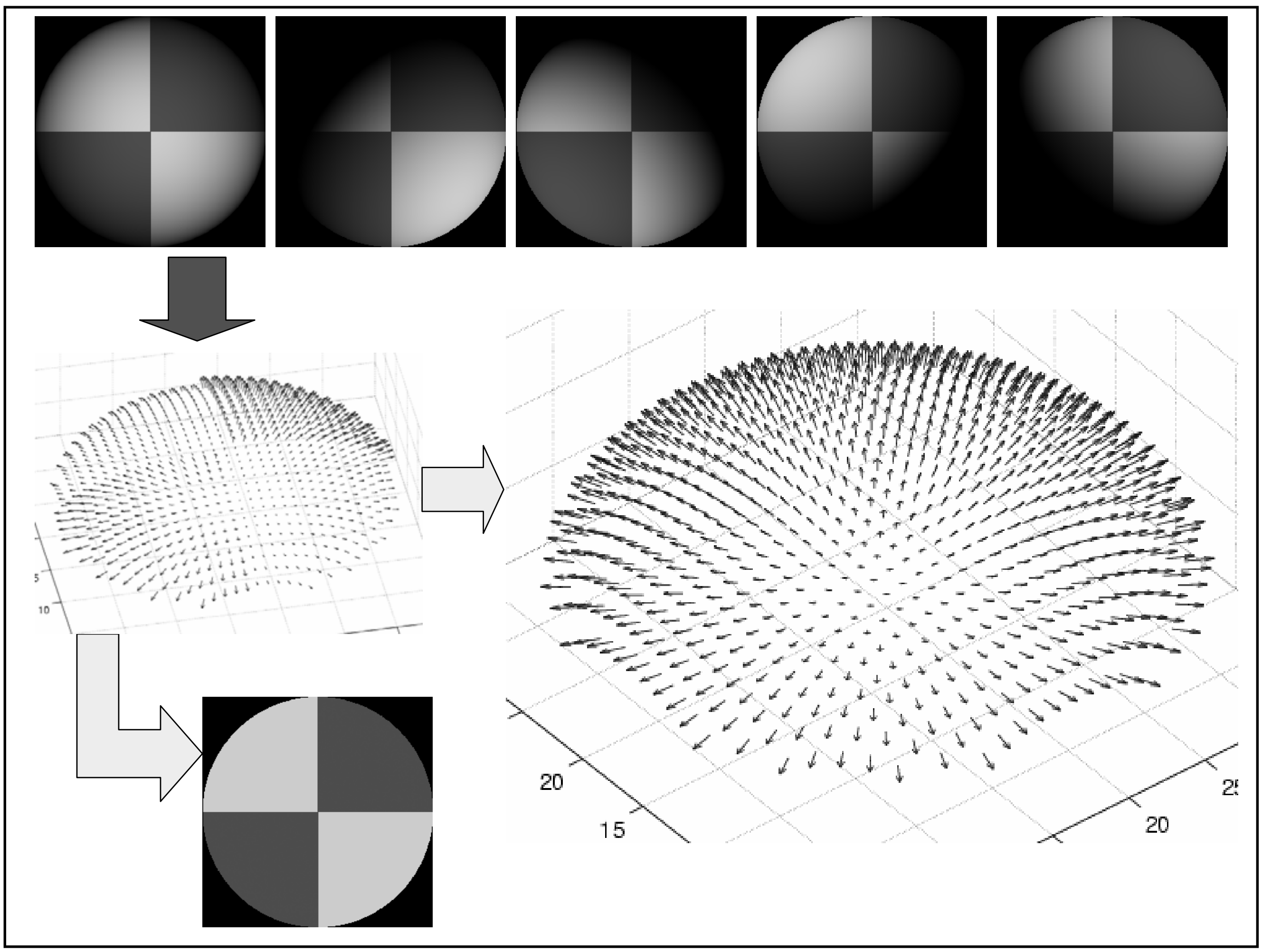
- Only works when there is no ambient illumination.


5
10

Photometric Stereo: Example (3)

$\mathbf{g}(P) = \rho(P)\mathbf{N}(P)$

$\rho(P) = |\, \mathbf{g}(P) \,|$

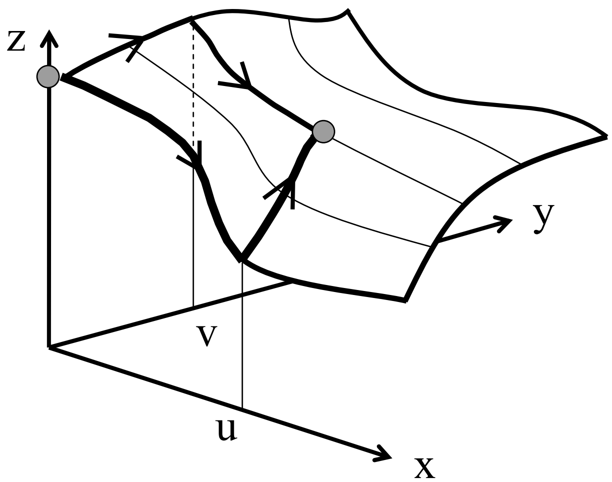


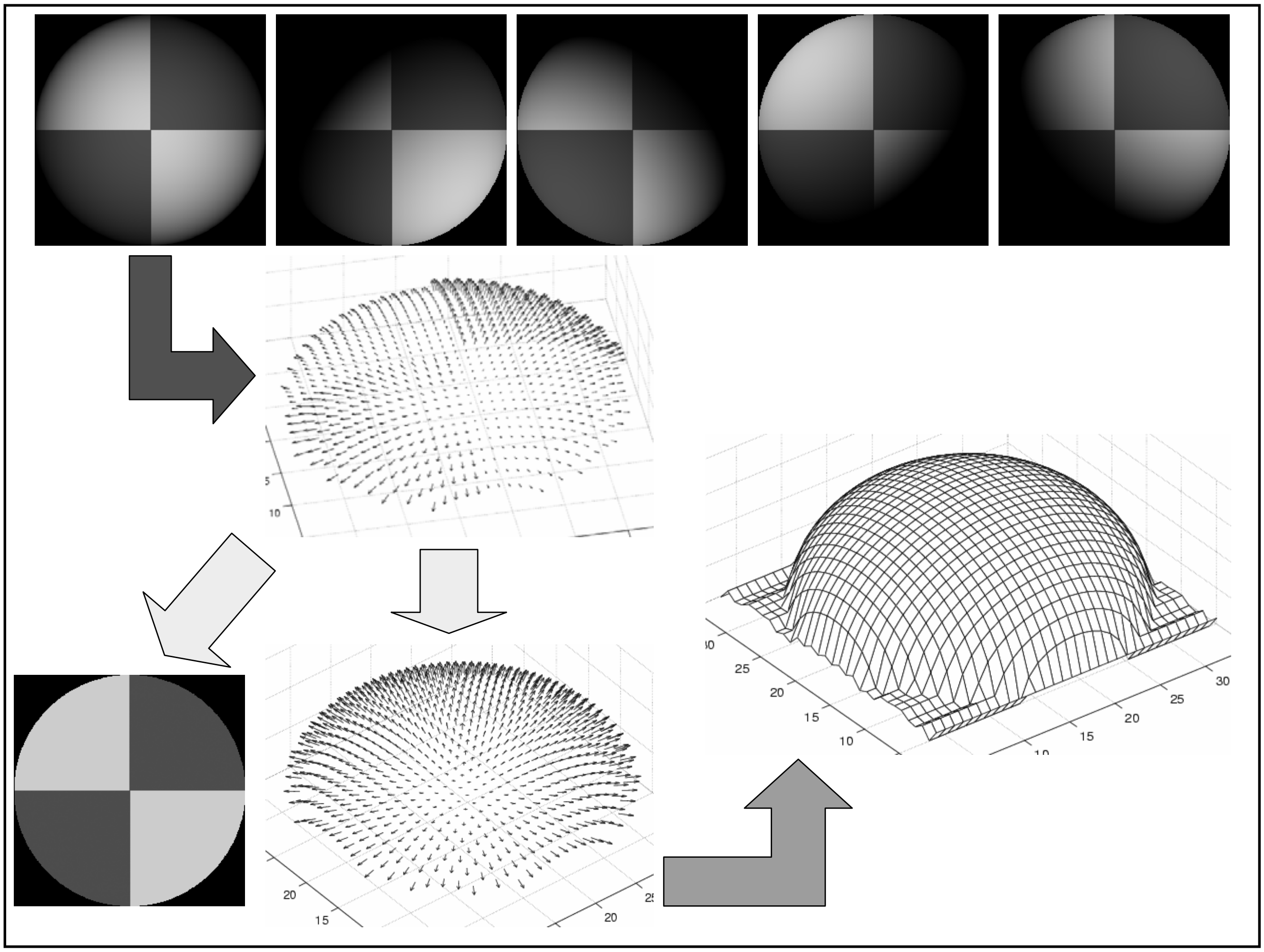
20
15
20

Photometric Stereo: Example (3)

Integrability!

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$





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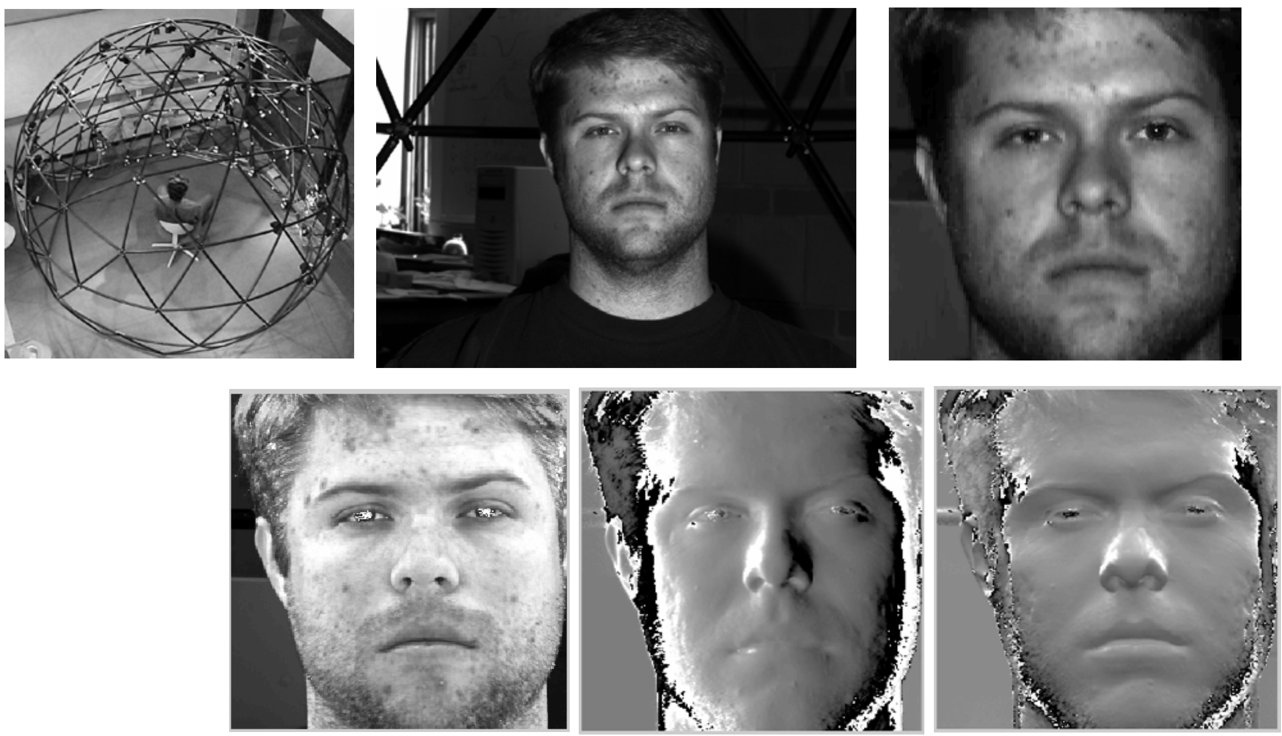
Photometric stereo example



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