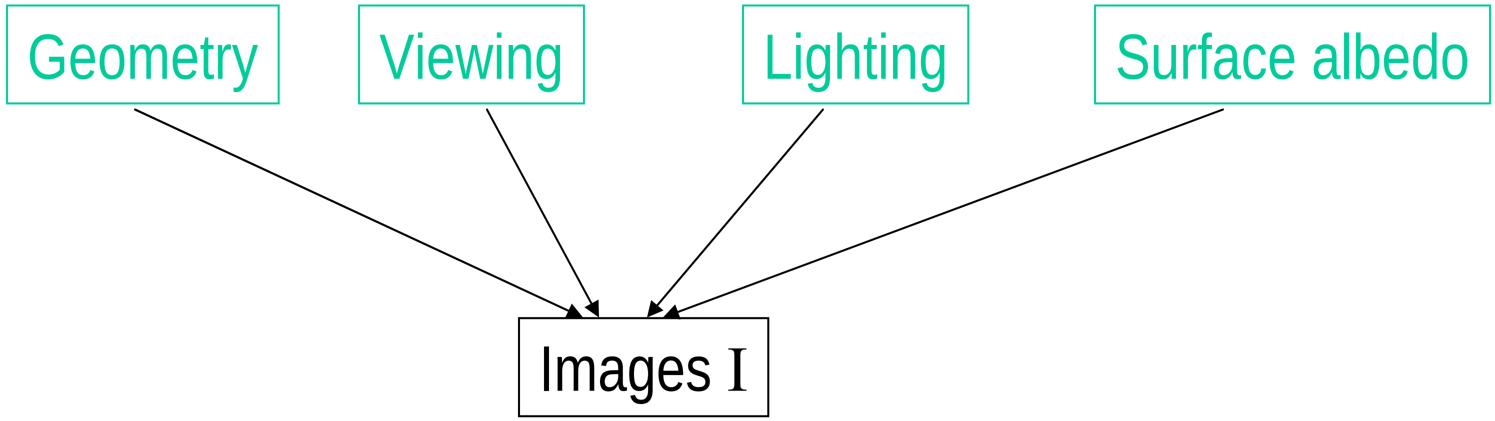
COS 429: COMPUTER VISON

RADIOMETRY (1 lecture)

- Elements of Radiometry
- Radiance
- Irradiance
- BRDF
- Photometric Stereo

Reading: Chapters 4 and 5

Many of the slides in this lecture are courtesy to Prof. J. Ponce


Geometry
Viewing
Lighting
Surface albedo
Images I

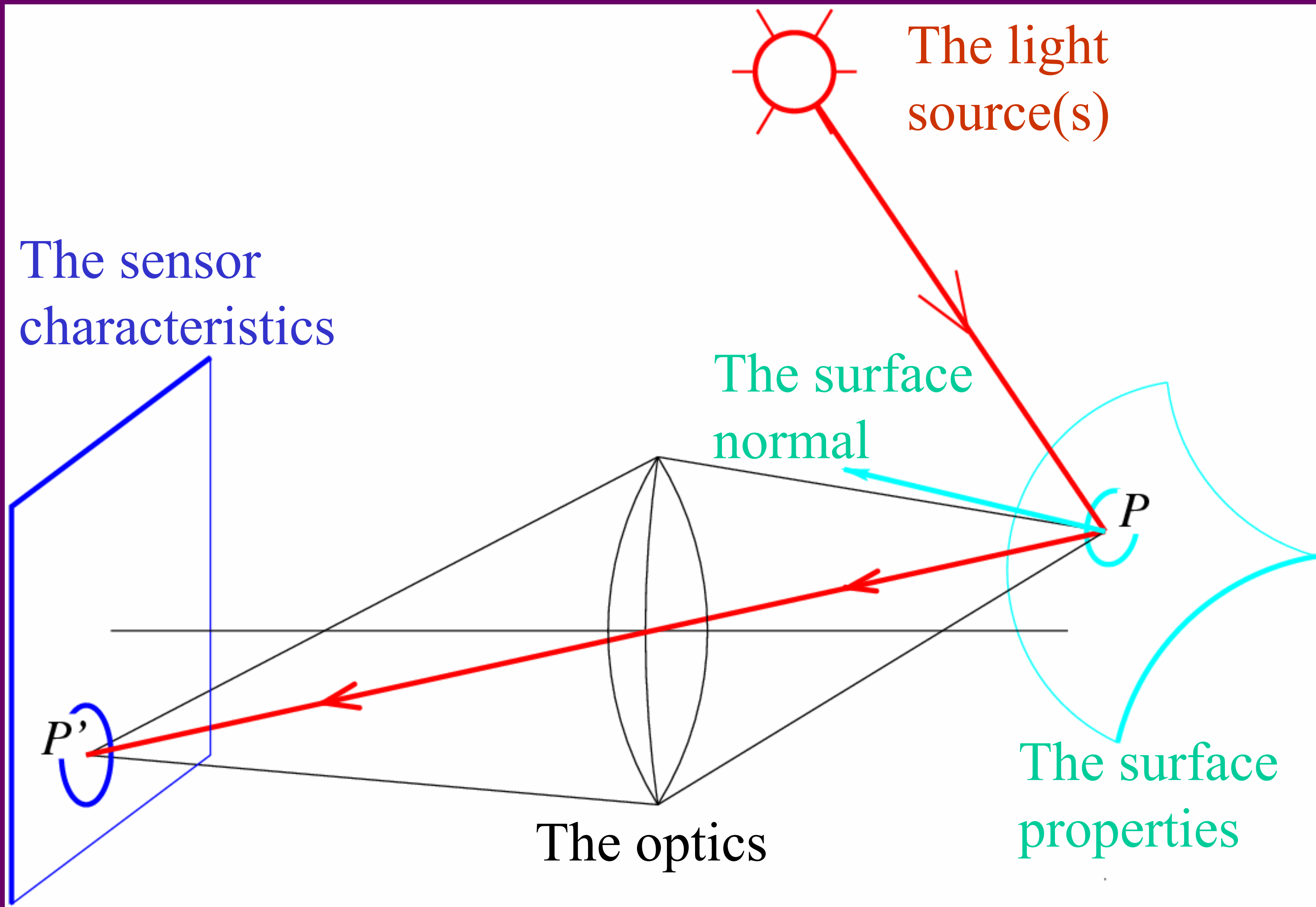
Photometric stereo example





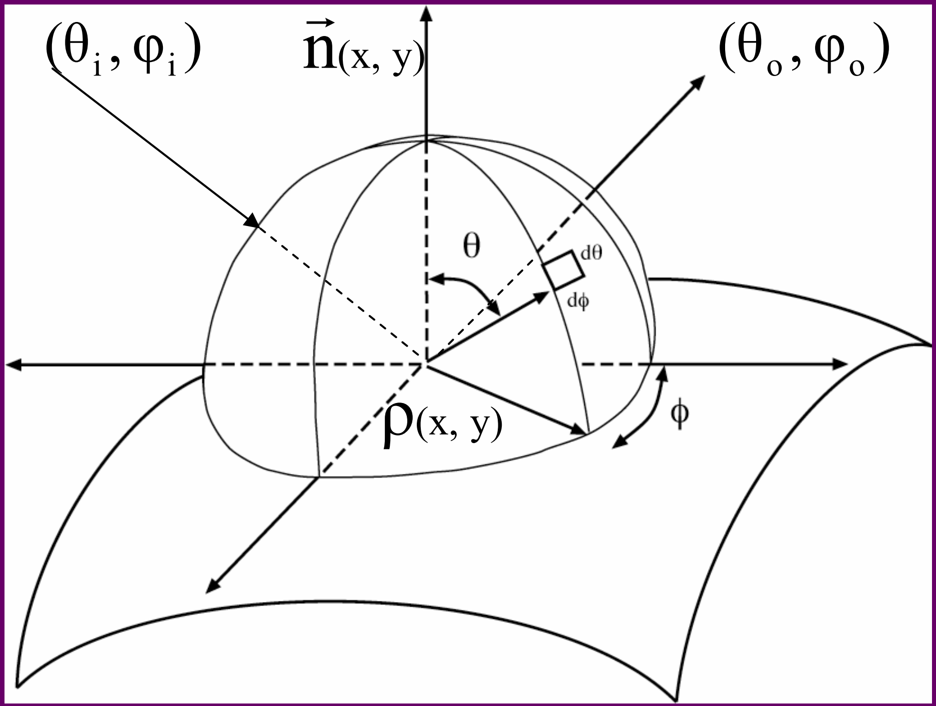

data from: http://www1.cs.columbia.edu/~belhumeur/pub/images/yalefacesB/readme

Image Formation: Radiometry



What determines the brightness of an image pixel?

The Illumination and Viewing Hemi-sphere



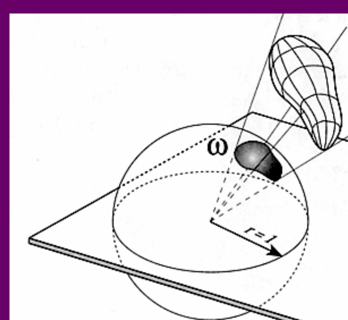
At infinitesimal, each point has a tangent plane, and thus a hemisphere Ω.

The ray of light is indexed by the polar coordinates

(θ, φ)

Foreshortening

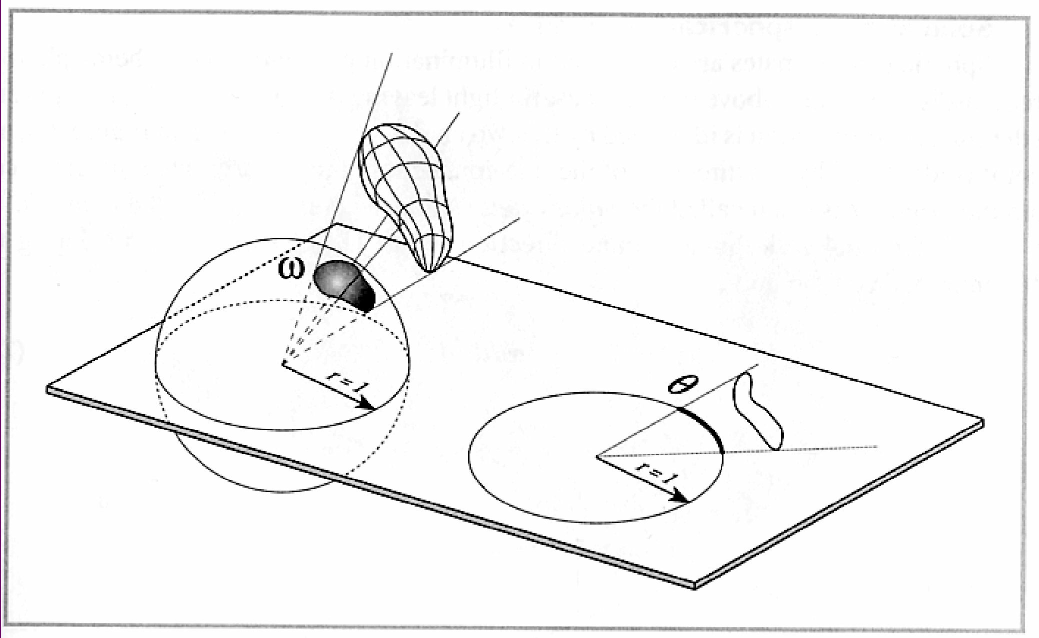
- **Principle:** two sources that look the same to a receiver must have the same effect on the receiver.
- **Principle:** two receivers that look the same to a source must receive the same amount of energy.
- "look the same" means produce the same input hemisphere (or output hemisphere)



- **Reason:** what else can a receiver know about a source but what appears on its input hemisphere? (ditto, swapping receiver and source)
- **Crucial consequence:** a big source (resp. receiver), viewed at a glancing angle, must produce (resp. experience) the same effect as a small source (resp. receiver) viewed frontally.

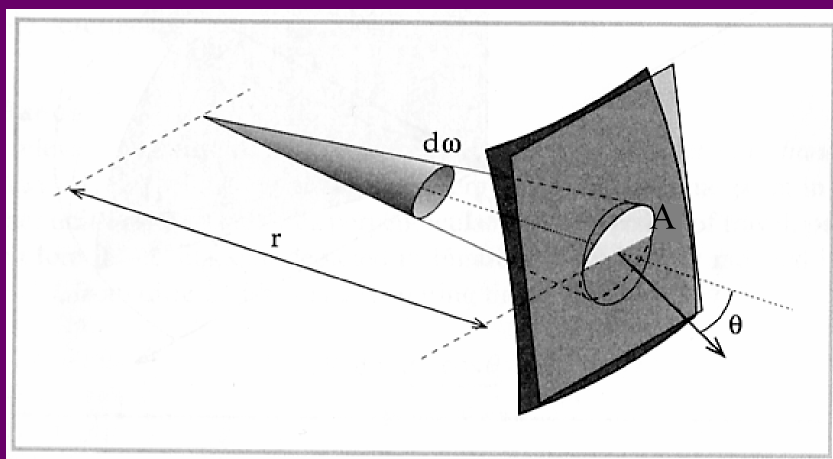
Measuring Angle

- To define radiance, we require the concept of solid angle
- **The solid angle sub-tended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P**
- Measured in *steradians*, sr
- Definition is analogous to projected angle in 2D
- If I'm at P, and I look out, solid angle tells me how much of my view is filled with an object



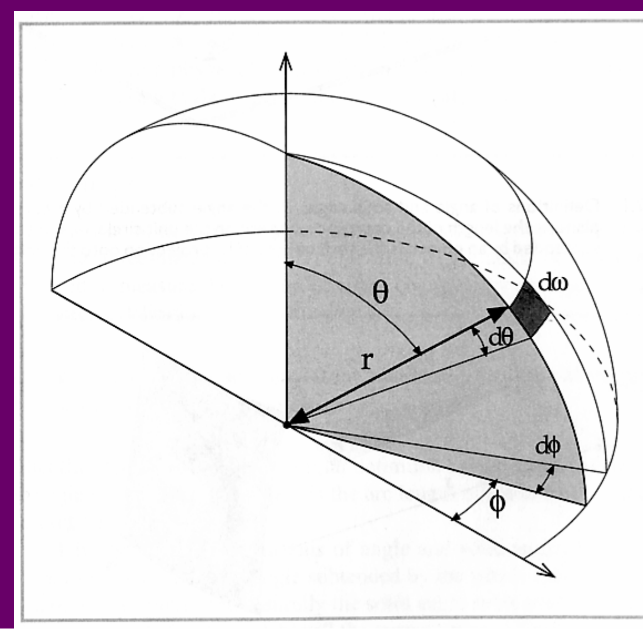
Solid Angle of a Small Patch

- Later, it will be important to talk about the solid angle of a small piece of surface

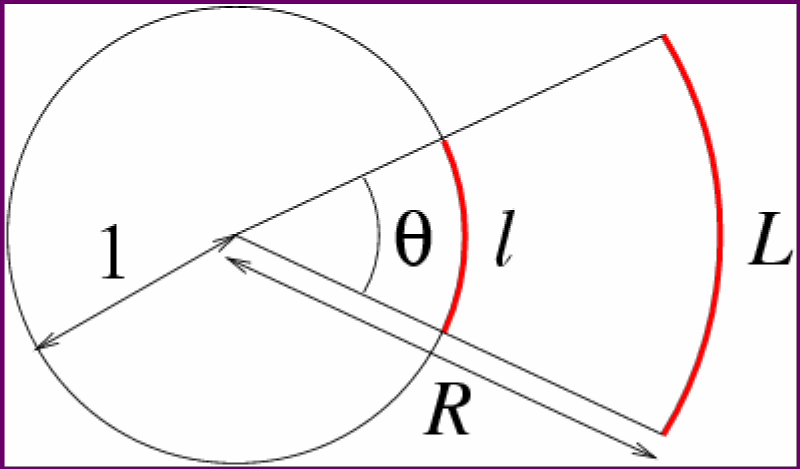


$$d\omega = \frac{dA\cos\theta}{r^2}$$

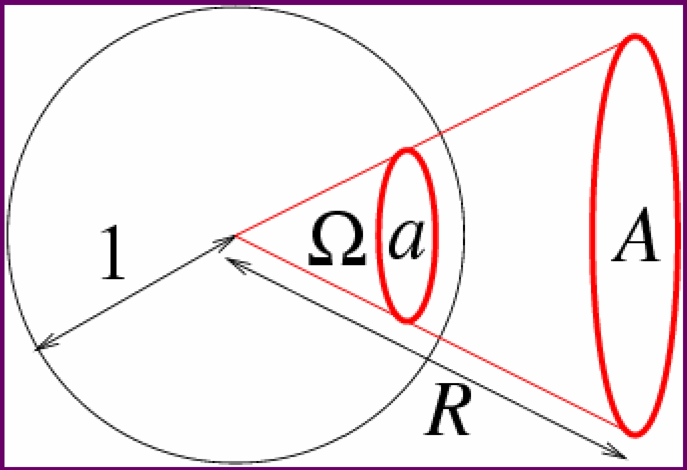
$$d\omega = \sin\theta d\theta d\phi$$



DEFINITION: Angles and Solid Angles

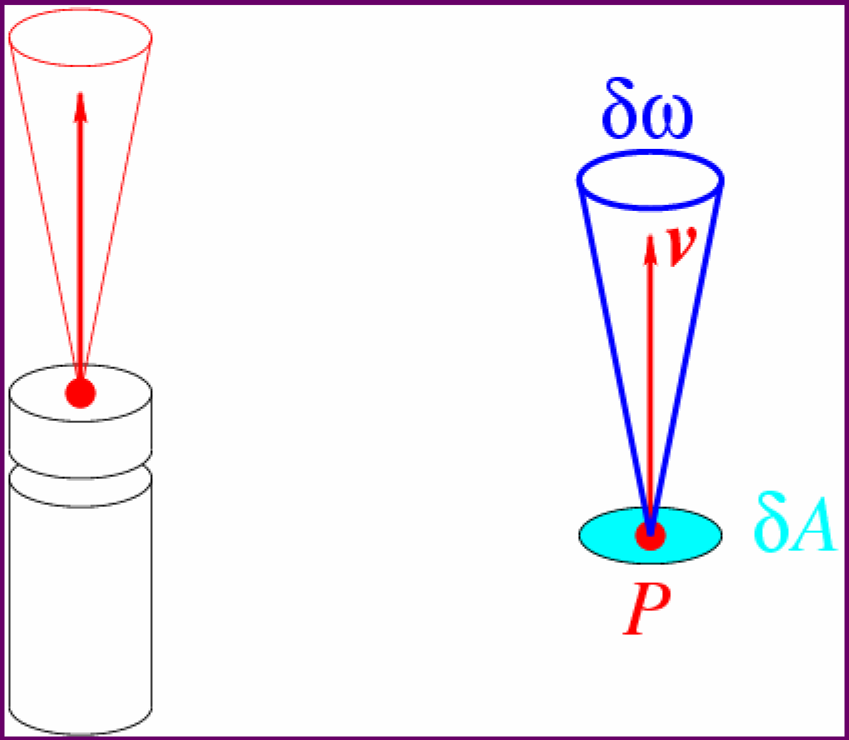


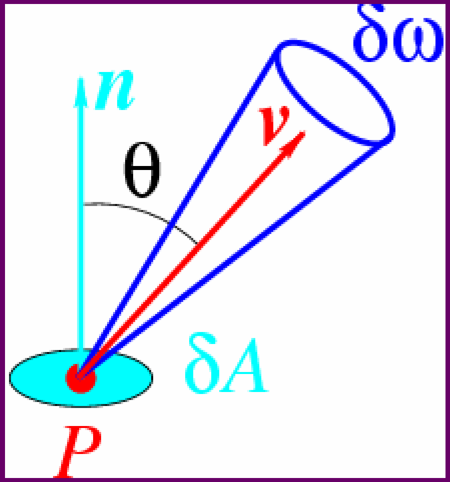
$$\theta = l = \frac{L}{R}$$ (radians)



$$\Omega = a = \frac{A}{R^2}$$ (steradians)

DEFINITION: The radiance is the power traveling at some point in a given direction per unit area perpendicular to this direction, per unit solid angle.

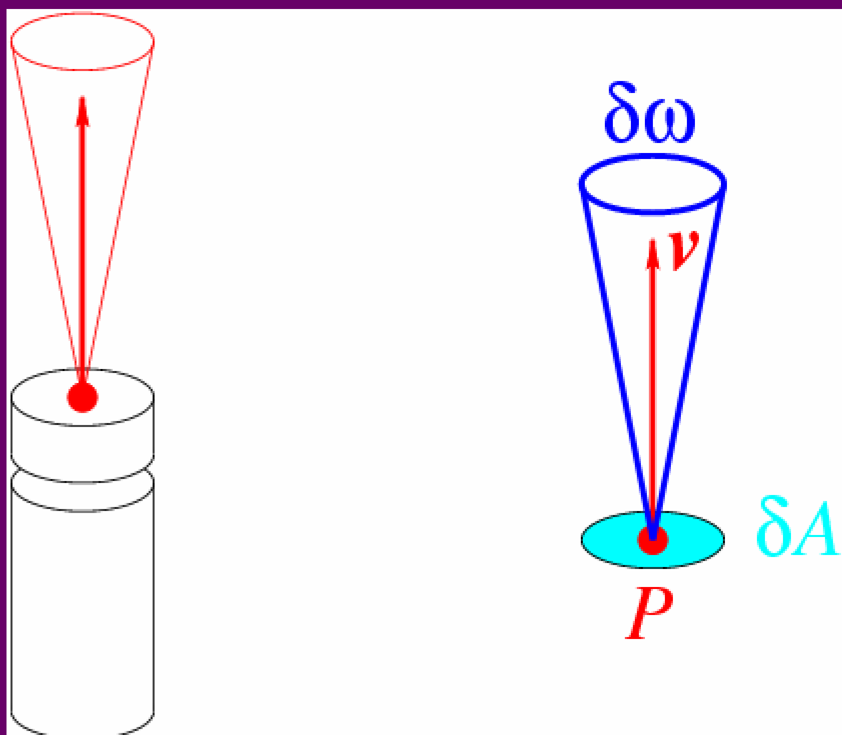


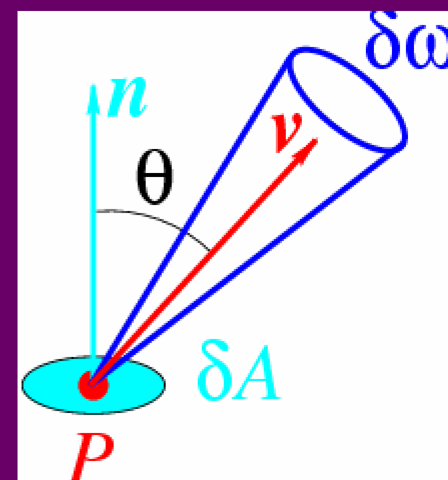


$\delta^2 P = L(P, \boldsymbol{v})\, \delta A\, \delta\omega$

$\delta^2 P = L(P, \boldsymbol{v}) \cos\theta\, \delta A\, \delta\omega$

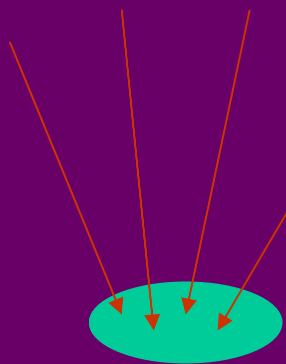
PROPERTY: Radiance is constant along straight lines (in vacuum).



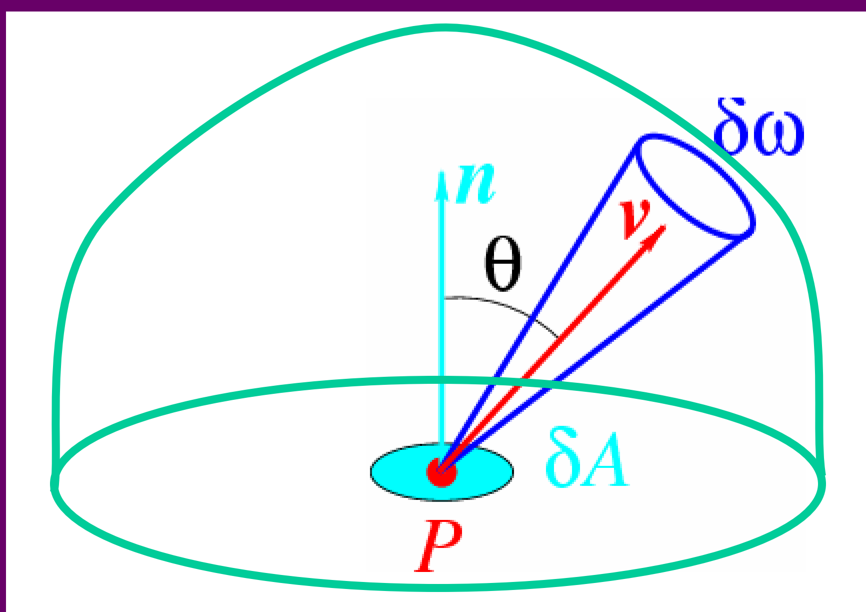


$$\delta^2 P = L(P, \boldsymbol{v})\, \delta A\, \delta\omega \quad \Rightarrow \quad \delta^2 P = L(P, \boldsymbol{v}) \cos\theta\, \delta A\, \delta\omega$$

DEFINITION: Irradiance



The irradiance is the power per unit area incident on a surface.

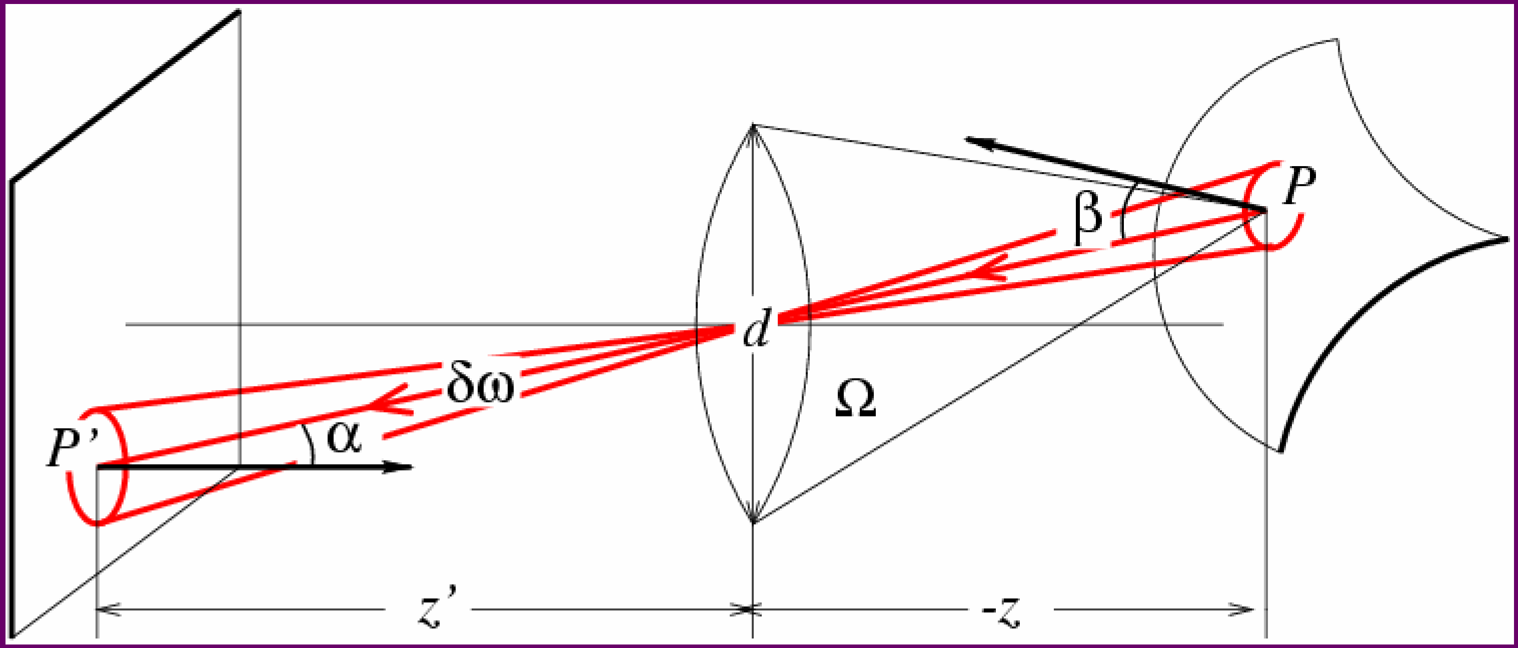


$$\delta^2 P = \delta E\ \delta A = L_i(\ P\ ,\ \boldsymbol{v}_i\ )\ \cos\theta_i \delta\omega_i\ \delta A$$

$$\delta E = L_i(\ P\ ,\ \boldsymbol{v}_i\ )\ \cos\theta_i\ \delta\omega_i$$

$$E = \int_H L_i\ (P,\ v_i)\ \cos\theta_i\ d\omega_i$$

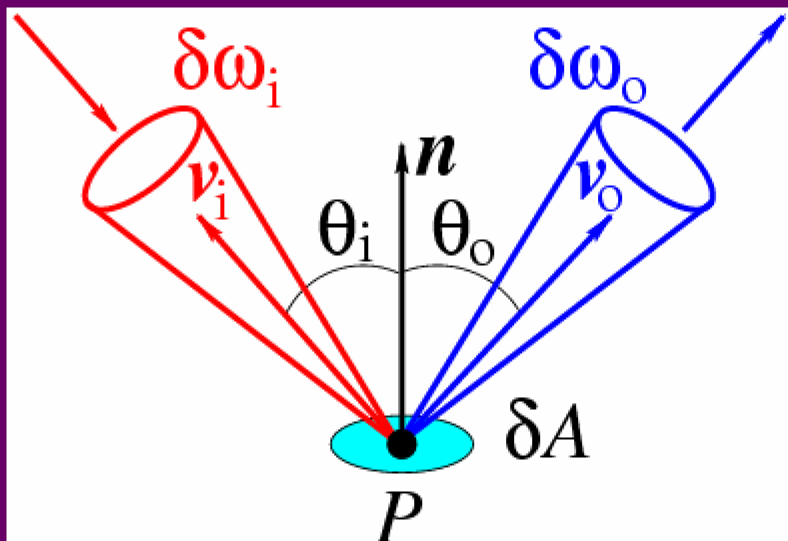
Photometry



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

- L is the radiance.
- E is the irradiance.

DEFINITION: The Bidirectional Reflectance Distribution Function (BRDF)



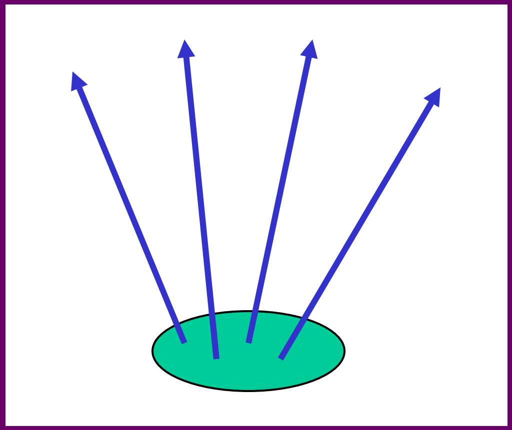
The BRDF is the ratio of the radiance in the outgoing direction to the incident irradiance (sr^{-1}).

$$L_o (P, \boldsymbol{v}_o) = \rho_{BD} (P, \boldsymbol{v}_i , \boldsymbol{v}_o) \, \delta E_i (P, \boldsymbol{v}_i)$$

$$= \rho_{BD} (P, \boldsymbol{v}_i , \boldsymbol{v}_o) \; L_i (P, \boldsymbol{v}_i) \cos \theta_i \; \delta\omega_i$$

Helmoltz reciprocity law: $\rho_{BD} (P, \boldsymbol{v}_i , \boldsymbol{v}_o) = \rho_{BD} (P, \boldsymbol{v}_o , \boldsymbol{v}_i)$

DEFINITION: Radiosity



The radiosity is the total power Leaving a point on a surface per unit area ($W * m^{-2}$).

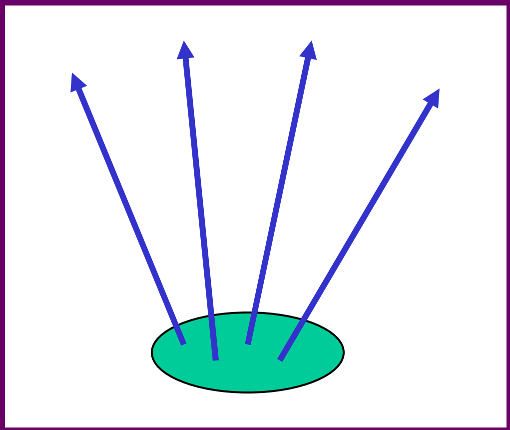
$$B(P) = \int_H L_o (P , \boldsymbol{v}_o) \cos\theta_o \, d\omega$$

Important case: L_o is independent of $\boldsymbol{v}_o$.

$$B(P) = \pi L_o(P)$$

DEFINITION: Lambertian (or Matte) Surfaces

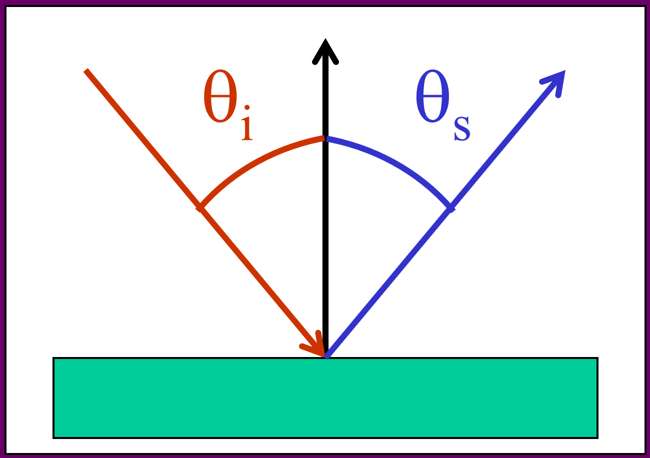
A Lambertian surface is a surface whose BRDF is independent of the outgoing direction (and by reciprocity of the incoming direction as well).



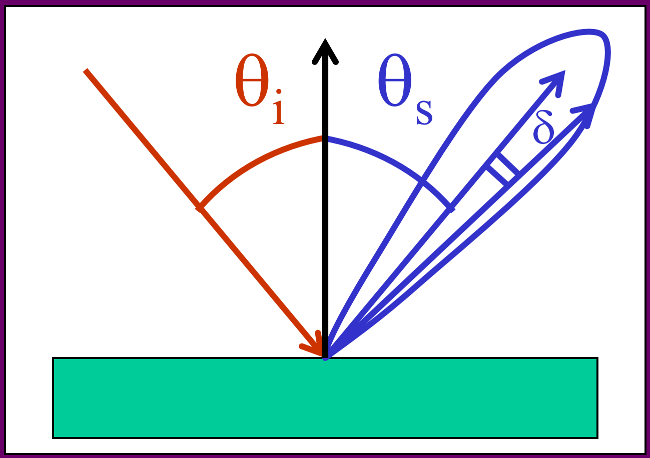
$$\rho_{BD}(\, \boldsymbol{v}_i \,, \boldsymbol{v}_o \,) = \rho_{BD} = \text{constant.}$$

The albedo is $\rho_d = \pi\, \rho_{BD}$.

DEFINITION: Specular Surfaces as Perfect or Rough Mirrors



Perfect mirror



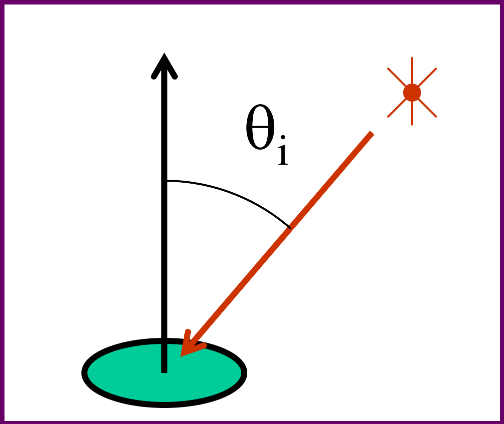
Rough mirror

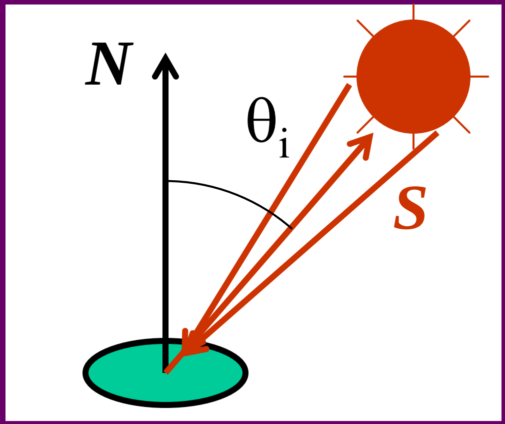
Perfect mirror: $L_o\ (P,\boldsymbol{v}_s) = L_i\ (P,\boldsymbol{v}_i)$

Phong (non-physical model): $L_o\ (P,\boldsymbol{v}_o)= \rho_s L_i(P,\boldsymbol{v}_i)\cos^n \delta$

Hybrid model: $L_o\ (P,\boldsymbol{v}_o)= \rho_d \int_H L_i(P,\boldsymbol{v}_i)\cos\theta_i\, d\omega_i + \rho_s L_i(P,\boldsymbol{v}_i)\cos^n \delta$

DEFINITION: Point Light Sources





A point light source is an idealization of an emitting sphere with radius ε at distance R, with ε << R and uniform radiance L_e emitted in every direction.

For a Lambertian surface, the corresponding radiosity is

$$B(P) = \left[\rho_d(P)\ L_e \frac{\pi \varepsilon^2}{R(P)^2} \right] \cos\theta_i \approx \rho_d(P) \frac{\mathbf{N}(P) \cdot \mathbf{S}(P)}{R(P)^2}$$

Local Shading Model

- Assume that the radiosity at a patch is the sum of the radiosities due to light source and sources alone.

No interreflections.

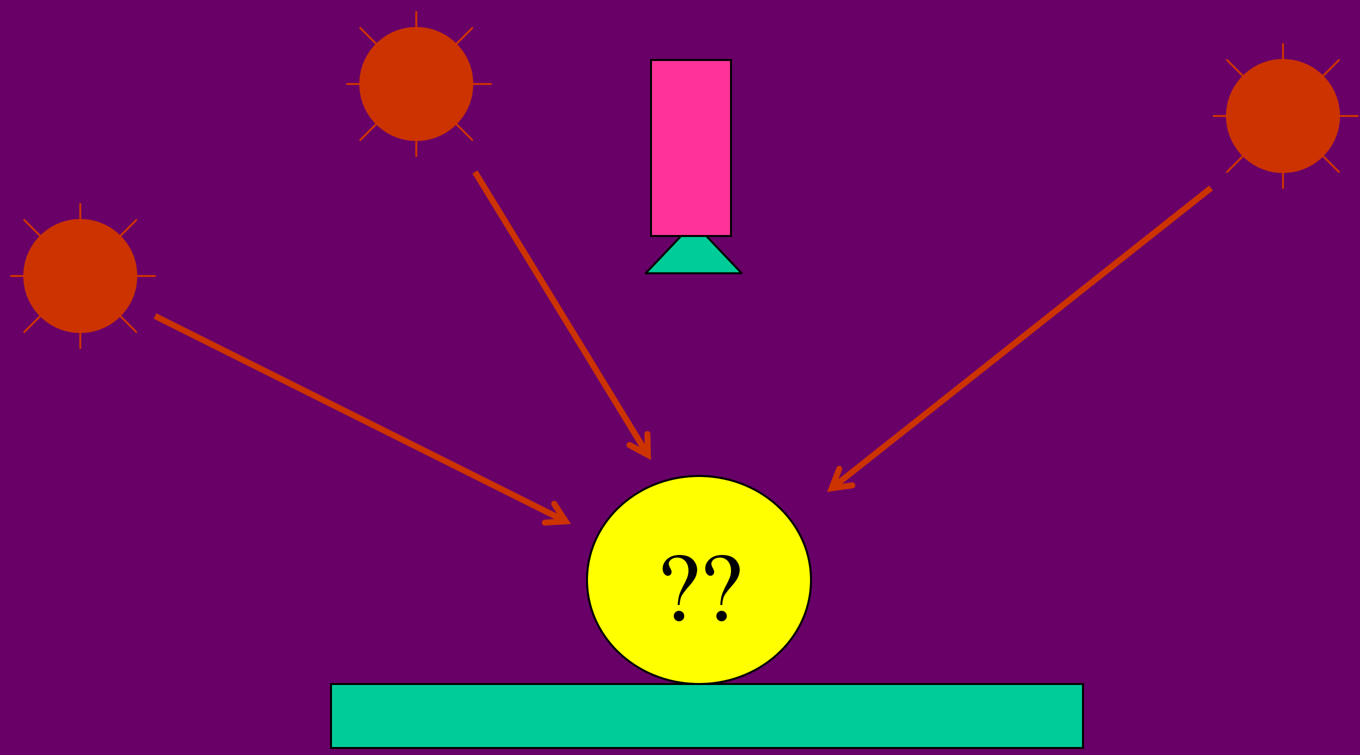
- For point sources:

$$B(P) = \sum_{\text{visible } s} \rho_d(P) \frac{\mathbf{N}(P) \cdot \mathbf{S}_s(P)}{R_s(P)^2}$$

- For point sources at infinity:

$$B(P) = \rho_d(P)\mathbf{N}(P) \cdot \sum_{\text{visible } s} \mathbf{S}_s(P)$$

Photometric Stereo (Woodham, 1979)



Problem: Given n images of an object, taken by a fixed camera under different (known) light sources, reconstruct the object shape.

Photometric Stereo: Example (1)

- Assume a Lambertian surface and distant point light sources.

$$I(P) = kB(P) = k\rho\,\mathbf{N}(P) \bullet \mathbf{S} = \mathbf{g}(P) \bullet \mathbf{V}$$

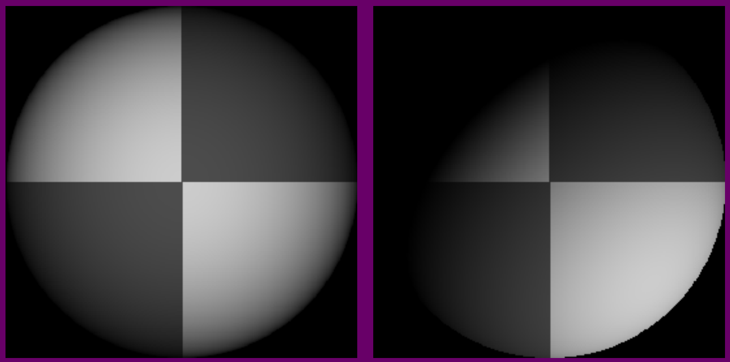
with $\mathbf{g}(P) = \rho\,\mathbf{N}(P)$ and $\mathbf{V} = k\,\mathbf{S}$

- Given n images, we obtain n linear equations in ***g***:

$$\mathbf{i} = \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \cdot \mathbf{g} \\ \mathbf{V}_2 \cdot \mathbf{g} \\ \dots \\ \mathbf{V}_n \cdot \mathbf{g} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \\ \mathbf{V}_n^T \end{bmatrix} \mathbf{g} \;\Rightarrow\; \mathbf{i} = \mathcal{V}\,\mathbf{g} \;\Rightarrow\; \mathbf{g} = \mathcal{V}^{-1}\,\mathbf{i}$$

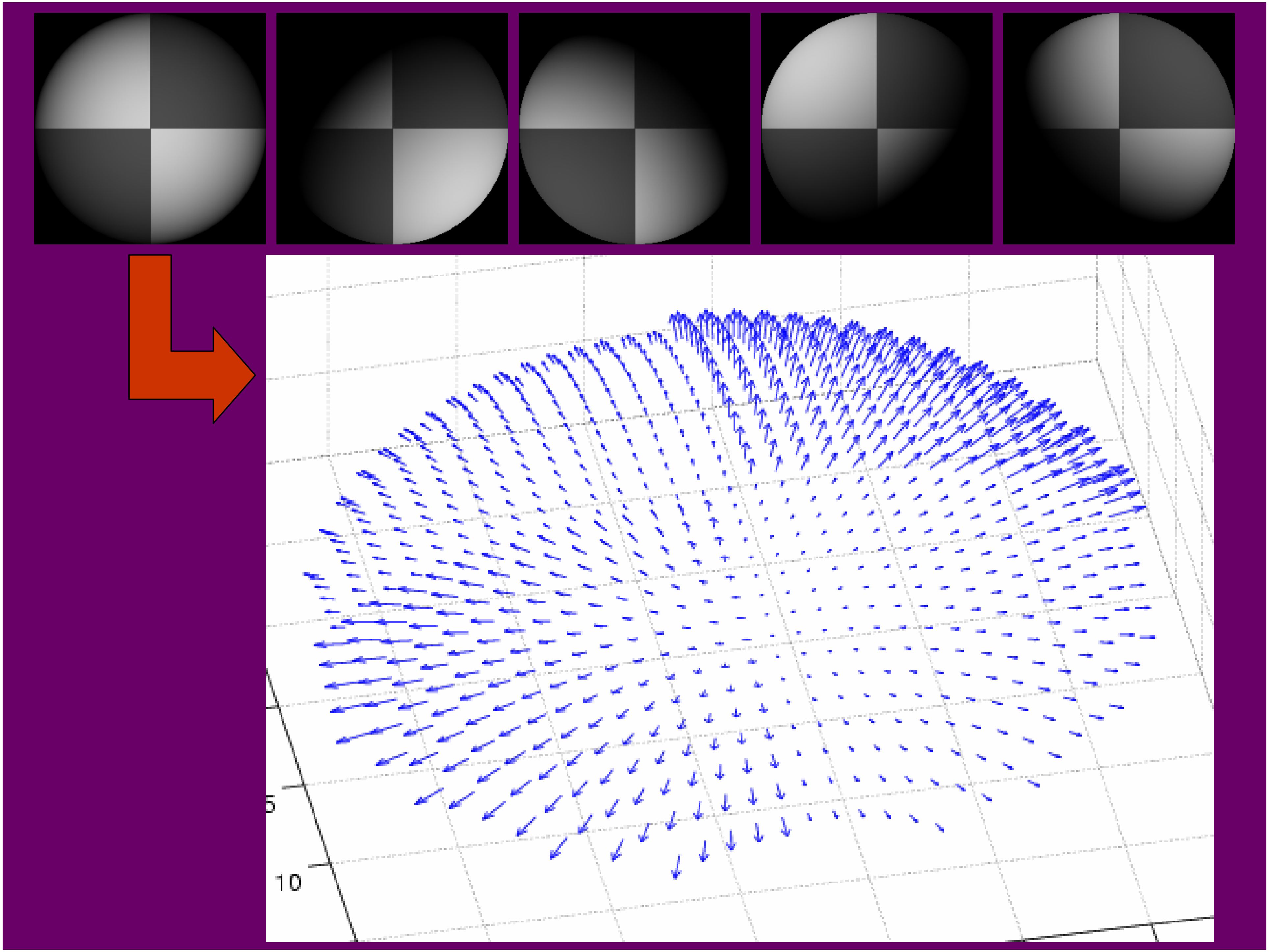
Photometric Stereo: Example (2)

- What about shadows?



- Just skip the equations corresponding to zero-intensity pixels.

- Only works when there is no ambient illumination.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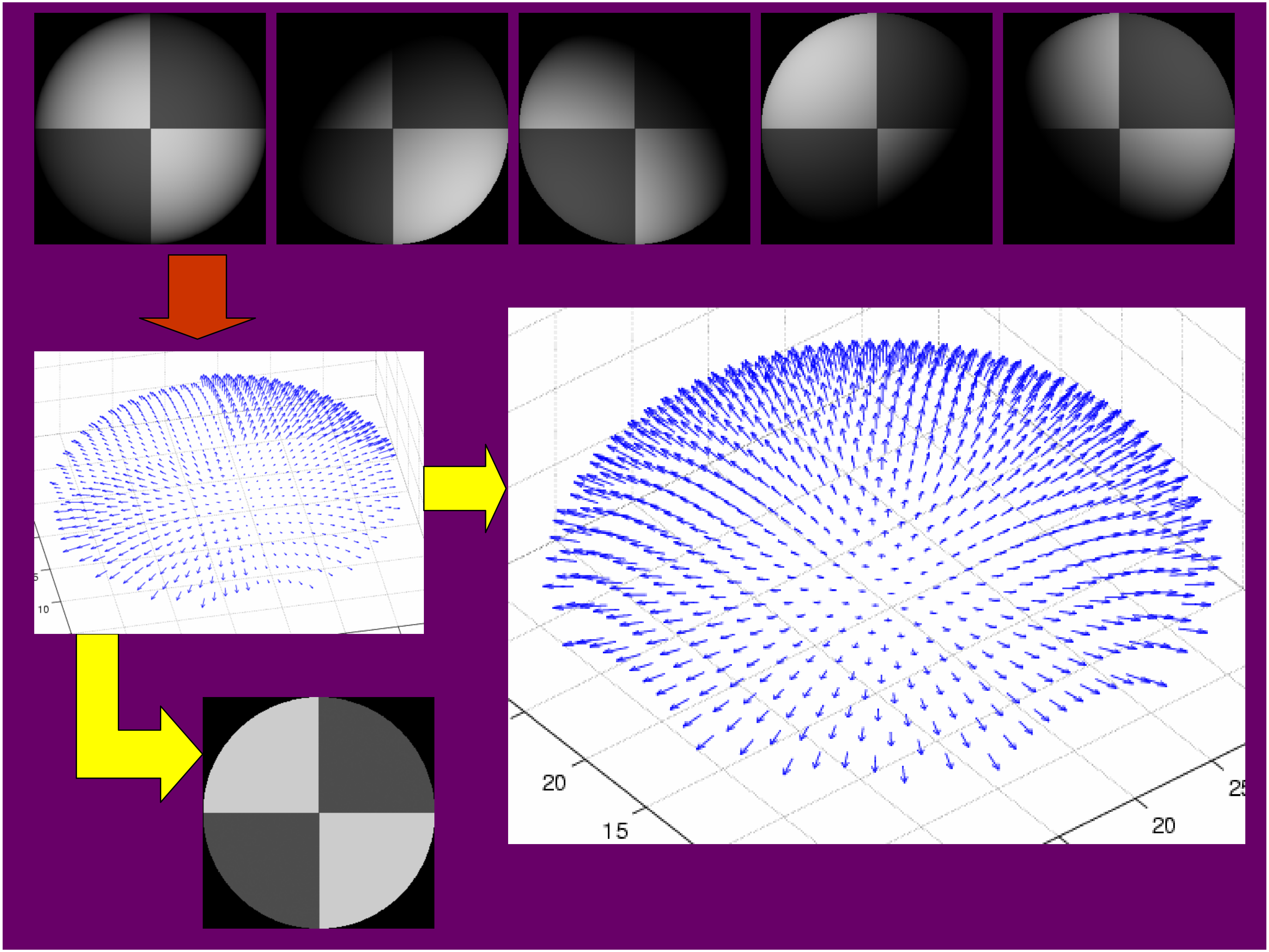
Photometric Stereo: Example (3)

$$\mathbf{g}(P) = \rho(P)\mathbf{N}(P)$$

$$\rho(P) = |\,\mathbf{g}(P)\,|$$

$$\mathbf{N}(P) = \frac{1}{|\,\mathbf{g}(P)\,|}\mathbf{g}(P)$$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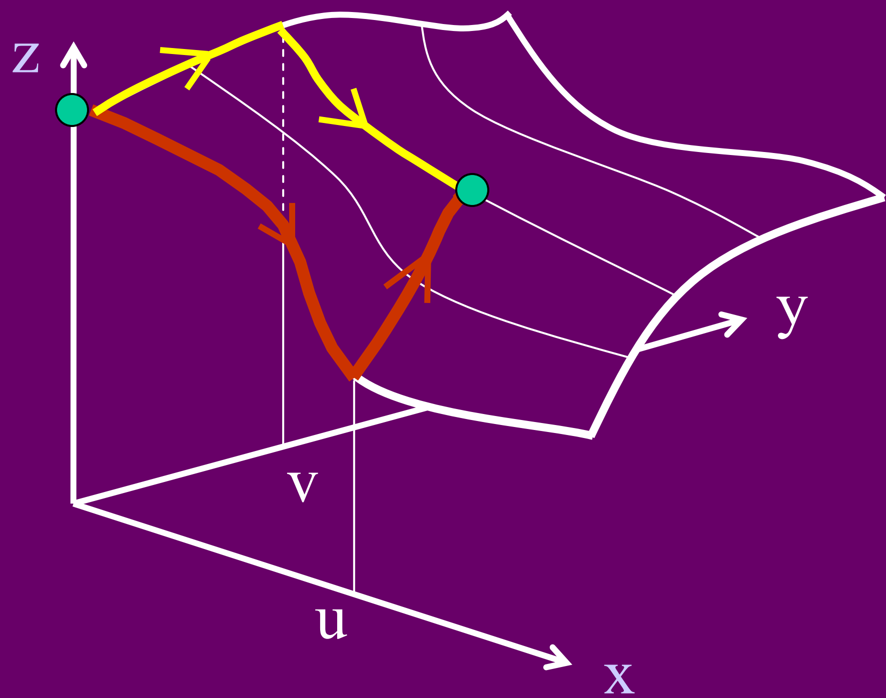
Photometric Stereo: Example (3)

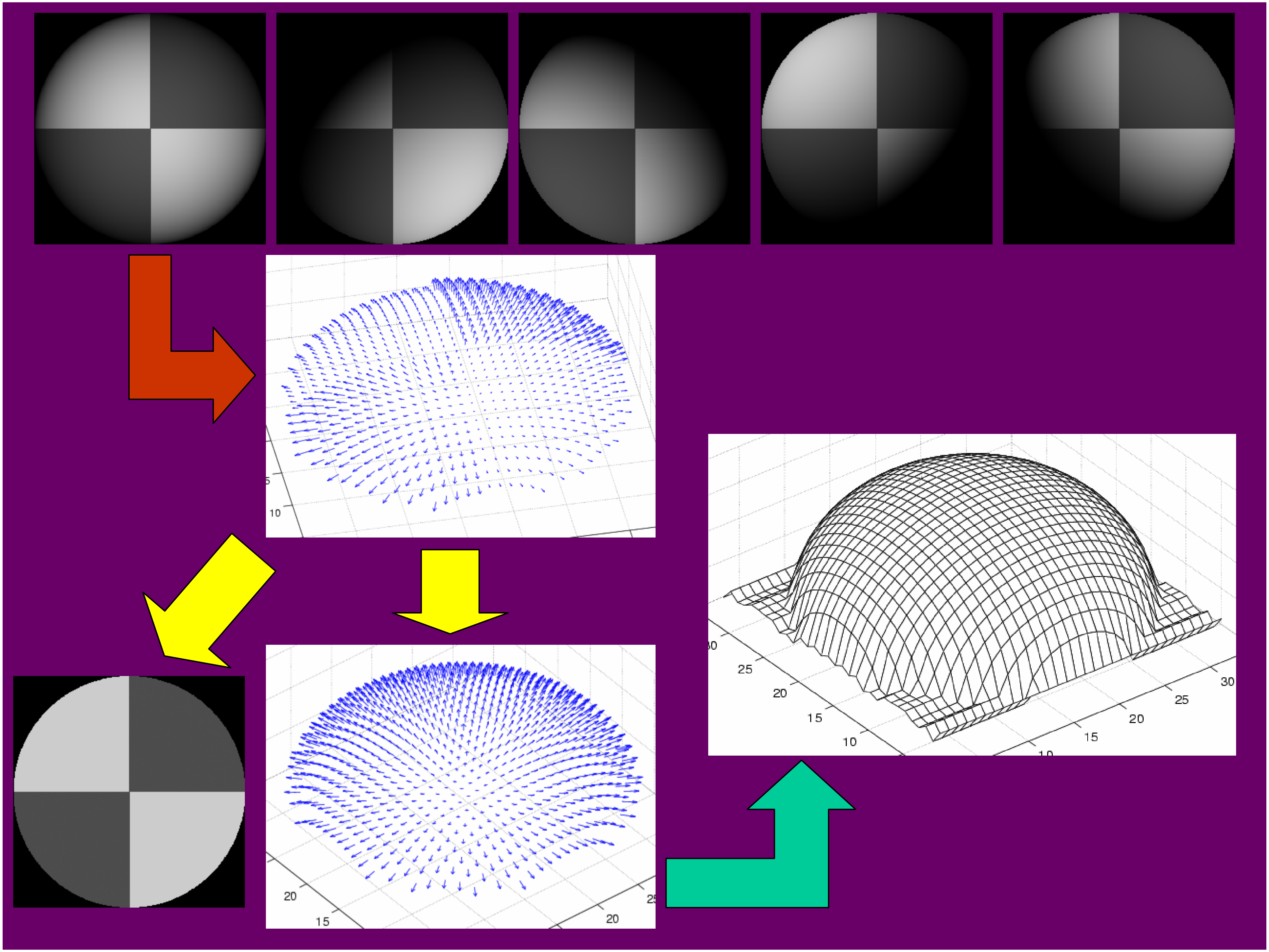
Integrability!

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$



$$\mathbf{N} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \propto \begin{bmatrix} -\frac{\partial z}{\partial x} \\ -\frac{\partial z}{\partial y} \\ 1 \end{bmatrix} \Rightarrow \begin{cases} \frac{\partial z}{\partial x} = -\frac{a}{c} \\ \frac{\partial z}{\partial y} = -\frac{b}{c} \end{cases}$$

$$z(u,v) = \int_0^u \frac{\partial z}{\partial x}(x,0)\,dx + \int_0^v \frac{\partial z}{\partial y}(u,y)\,dy$$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Photometric stereo example

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