Hash Tables

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COS 217

Goals of Today's Lecture

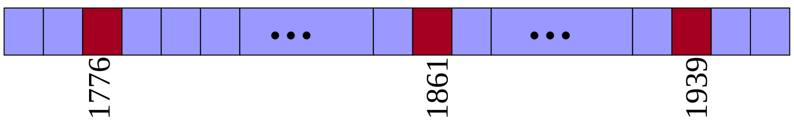
- Motivation for hash tables
  - Examples of (key, value) pairs
  - Limitations of using arrays
  - Example using a linked list
  - Inefficiency of using a linked list
- Hash tables
  - Hash table data structure
  - Hash function
  - Example hashing code
  - Who owns the keys?
- Implementing "mod" efficiently
  - Binary representation of numbers
  - Logical bit operators

Accessing Data By a Key

- Student grades: (name, grade)
  - E.g., ("john smith", 84), ("jane doe", 93), ("bill clinton", 81)
  - Gradeof("john smith") returns 84
  - Gradeof("joe schmoe") returns NULL
- Wine inventory: (name, #bottles)
  - E.g., ("tapestry", 3), ("latour", 12), ("margeaux", 3)
  - Bottlesof("latour") returns 12
  - Bottlesof("giesen") returns NULL
- Years when a war started: (year, war)
  - E.g., (1776, "Revolutionary"), (1861, "Civil War"), (1939, "WW2")
  - Warstarted(1939) returns "WW2"
  - Warstarted(1984) returns NULL
- Symbol table: (variable name, variable value)
  - E.g., ("MAXARRAY", 2000), ("FOO", 7), ("BAR", -10)

Limitations of Using an Array

- Array stores n values indexed 0, …, n-1
  - Index is an integer
  - Max size must be known in advance
- But, the key in a (key, value) pair might not be a number
  - Well, could convert it to a number
    - E.g., have a separate number for each possible name
- But, we'd need an extremely large array
  - Large number of possible keys (e.g., all names, all years, etc.)
  - And, the number of unique keys might even be unknown
  - And, most of the array elements would be empty



Could Use an Array of (key, value)

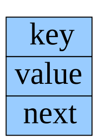
- Alternative way to use an array
  - Array element i is a struct that stores key and value

0	1776	Revolutionary
1	1861	Civil
2	1939	WW2

- Managing the array
  - Add an elements: add to the end
  - Remove an element: find the element, and copy last element over it
  - Find an element: search from the beginning of the array
- Problems
  - Allocating too little memory: run out of space
  - Allocating too much memory: wasteful of space

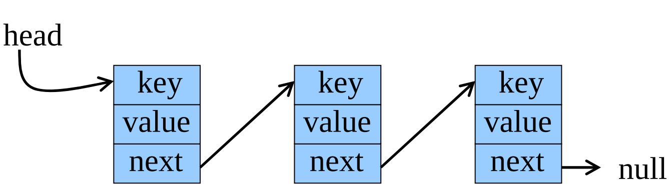
Linked List to Adapt Memory Size

- Each element is a struct
  - Key
  - Value
  - Pointer to next element



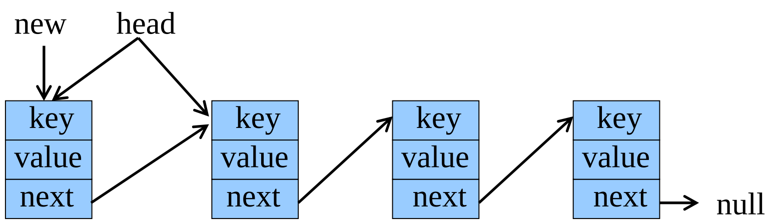

```
struct Entry {
  int key;
  char* value;
  struct Entry *next;
};
```

- Linked list
  - Pointer to the first element in the list
  - Functions for adding and removing elements
  - Function for searching for an element with a particular key



Adding Element to a List

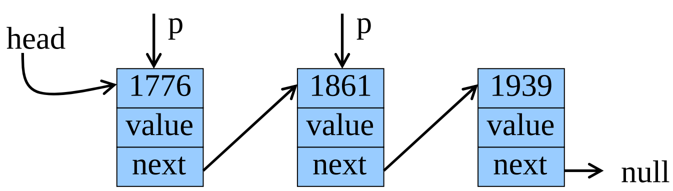
- Add new element at front of list
  - Make ptr of new element point the current first element
    - `new->next = head;`
  - Make the head of the list point to the new element
    - `head = new;`



Locating an Element in a List

- Sequence through the list by key value
  - Return pointer to the element
  - … or NULL if no element is found

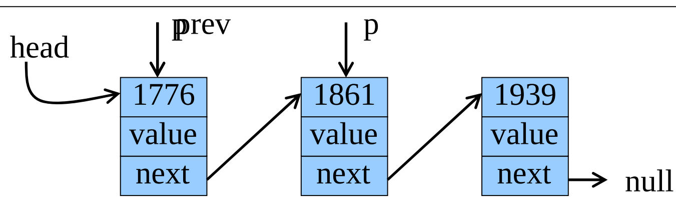
```
for (p = head; p!=NULL; p=p->next) {
   if (p->key == 1861)
      return p;
}
return NULL;
```



Locate and Remove an Element (1)

- Sequence through the list by key value
  - Keep track of the previous element in the list

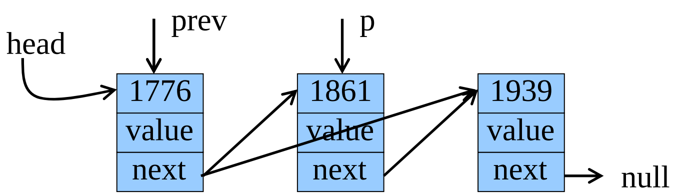
```
prev = NULL;
for (p = head; p!=NULL; prev=p, p=p->next){
   if (p->key == 1861) {
      delete the element (see next slide!);
      break;
   }
}
```



Locate and Remove an Element (2)

- Delete the element
  - Head element: make head point to the second element
  - Non-head element: make previous Entry point to next element

```
if (p == head)
   head = head->next;
else
   prev->next = p->next;
```

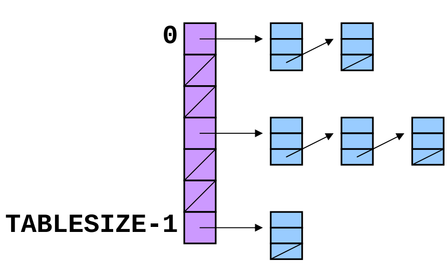


List is Not Good for (key, value)

- Good place to start
  - Simple algorithm and data structure
  - Good to allow early start on design and test of client code
- But, testing might show that this is not efficient enough
  - Removing or locating an element
    - Requires walking through the elements in the list
  - Could store elements in sorted order
    - But, keeping them in sorted order is time consuming
    - And, searching by key in the sorted list still takes time
- Ultimately, we need a better approach
  - Memory efficient: adds extra memory as needed
  - Time efficient: finds element by its key instantly (or nearly)

Hash Table

- Fixed-size array where each element points to a linked list

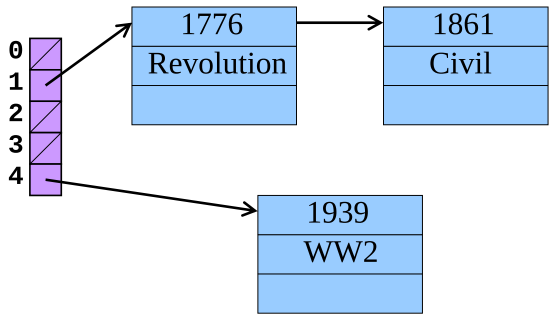



```
struct Entry *hashtab[TABLESIZE];
```

- Function mapping each key to an array index
  - For example, for an integer key **h**
    - Hash function: **i = h % TABLESIZE** (mod function)
  - Go to array element **i**, i.e., the linked list **hashtab[i]**
    - Search for element, add element, remove element, etc.

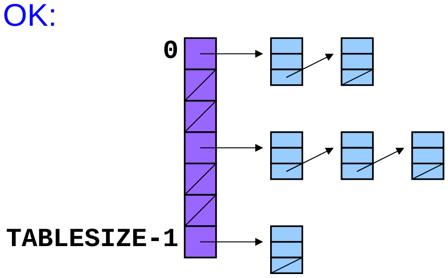
Example

- Array of size 5 with hash function “h mod 5”
  - “1776 % 5” is 1
  - “1861 % 5” is 1
  - “1939 % 5” is 4



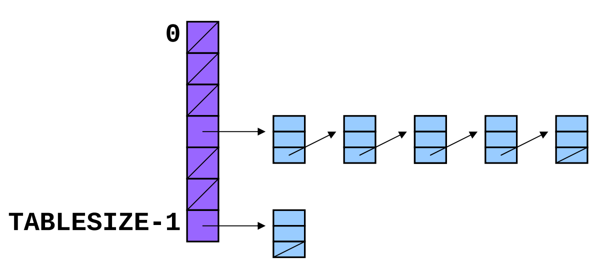
How Large an Array?

- Large enough that average “bucket” size is 1
  - Short buckets mean fast look-ups
  - Long buckets mean slow look-ups
- Small enough to be memory efficient
  - Not an excessive number of elements
  - Fortunately, each array element is just storing a pointer
- This is OK:



What Kind of Hash Function?

- Good at distributing elements across the array
  - Distribute results over the range 0, 1, …, TABLESIZE-1
  - Distribute results *evenly* to avoid very long buckets
- This is not so good:



Hashing String Keys to Integers

- Simple schemes don't distribute the keys evenly enough
  - Number of characters, mod TABLESIZE
  - Sum the ASCII values of all characters, mod TABLESIZE
  - …
- Here's a reasonably good hash function
  - Weighted sum of characters $\mathbf{x_i}$ in the string
    - $(\Sigma\ a^i x_i)$ mod TABLESIZE
  - Best if a and TABLESIZE are relatively prime
    - E.g., a = 65599, TABLESIZE = 1024

Implementing Hash Function

- Potentially expensive to compute a^i for each value of i
  - Computing a^i for each value of I
  - Instead, do (((x[0] * 65599 + x[1]) * 65599 + x[2]) * 65599 + x[3]) * …

```
unsigned hash(char *x) {
  int i; unsigned int h = 0;
  for (i=0; x[i]; i++)
      h = h * 65599 + x[i];
  return (h % 1024);
}
```

Can be more clever than this for powers of two!

Hash Table Example

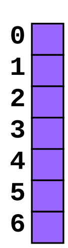
Example: TABLESIZE = 7

Lookup (and enter, if not present) these strings: the, cat, in, the, hat

Hash table initially empty.

First word: the. hash("the") = 965156977. 965156977 % 7 = 1.

Search the linked list table[1] for the string "the"; not found.



Hash Table Example

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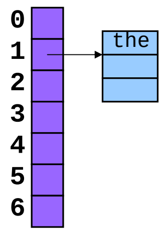
Search the linked list table[1] for the string "the"; not found

Now: table[1] = makelink(key, value, table[1])

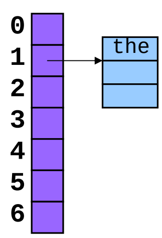


Hash Table Example

Second word: "cat". hash("cat") = 3895848756. 3895848756 % 7 = 2.

Search the linked list table[2] for the string "cat"; not found

Now: table[2] = makelink(key, value, table[2])

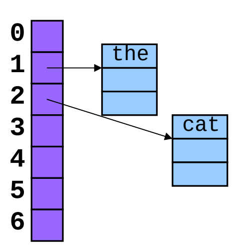


Hash Table Example

Third word: "in". hash("in") = 6888005. 6888005% 7 = 5.

Search the linked list table[5] for the string "in"; not found

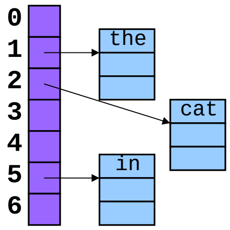
Now: table[5] = makelink(key, value, table[5])



Hash Table Example

Fourth word: “the”. hash(“the”) = 965156977. 965156977 % 7 = 1.

Search the linked list table[1] for the string “the”; found it!



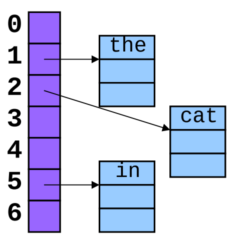
Hash Table Example

Fourth word: “hat”. hash(“hat”) = 865559739. 865559739 % 7 = 2.

Search the linked list table[2] for the string “hat”; not found.

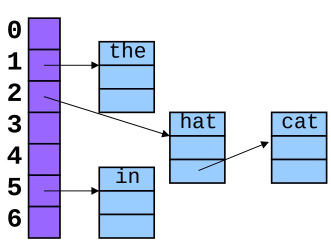
Now, insert “hat” into the linked list table[2].

At beginning or end? Doesn’t matter.



Hash Table Example

Inserting at the front is easier, so add “hat” at the front



Example Hash Table C Code

- Element in the hash table

```
struct Nlist {
  struct Nlist *next;
  char *key;
  char *value;
};
```

- Hash table
  - `struct Nlist *hashtab[1024];`
- Three functions
  - Hash function: `unsigned hash(char *x)`
  - Look up with key: `struct Nlist *lookup(char *s)`
  - Install entry: `struct Nlist *install(char *key, *value)`

Lookup Function

- Lookup based on key
  - Key is a string *s
  - Return pointer to matching hash-table element
  - … or return NULL if no match is found

```
struct Nlist *lookup(char *s) {
   struct Nlist *p;

   for (p = hashtab[hash(s)]; p!=NULL; p=p->next)
      if (strcmp(s, p->key) == 0)
         return p; /* found */
   return NULL;    /* not found */
}
```


Install an Entry (1)

- Install and (key, value) pair
  - Add new Entry if none exists, or overwrite the old value
  - Return a pointer to the Entry

```
struct Nlist *install(char *key, char *value) {
   struct Nlist *p;

   if ((p = lookup(name)) == NULL) { /* not found */
      create and add new Entry (see next slide);
   } else  /* already there, so discard old value */
      free(p->value);
   p->value = malloc(strlen(value) + 1);
   assert(p->value != NULL);
   strcpy(p->value, value);
   return p;
}
```

Install an Entry (2)

- Create and install a new Entry
  - Allocate memory for the new struct and the key
  - Insert into the appropriate linked list in the hash table

```
p = malloc(sizeof(*p));
assert(p != NULL);
p->key = malloc(strlen(key) + 1);
assert(p->key != NULL);
strcpy(p->key, key);

/* add to front of linked list */
unsigned hashval = hash(key);
p->next = hashtab[hashval]
hashtab[hashval] = p;
```


Why Bother Copying the Key?

- In the example, why did I do

```
p->key = malloc(strlen(key) + 1);
strcpy(p->key, key);
```

- Instead of simply

```
p->key = key;
```

- After all, the client passed me **key**, which is a *pointer*
  - So, storage for the key has already been allocated
  - Don't I simply need to copy the *address* where the string is stored?
- I want to preserve the integrity of the hash table
  - Even if the client program ultimately "frees" the memory for key
  - So, the install function makes a copy of the key
- The hash table *owns* the key
  - … because it is part of the data structure

Revisiting Hash Functions

- Potentially expensive to compute "mod c"
  - Involves division by c and keeping the remainder
  - Easier when c is a power of 2 (e.g., 16 = 2^4)
- Binary (base 2) representation of numbers
  - E.g., 53 = 32 + 16 + 4 + 1

•••	32	16	8	4	2	1	
0	0	1	1	0	1	0	1

  - E.g., 53 % 16 is 5, the last four bits of the number

•••	32	16	8	4	2	1	
0	0	0	0	0	1	0	1

  - Would like an easy way to isolate the last four bits…

Bitwise Operators in C

- Bitwise AND (&)

&	0	1
0	0	0
1	0	1

  - Mod on the cheap!
    - E.g., h = 53 & 15;

53	0	0	1	1	0	1	0	1
& 15	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	0	1

- Bitwise OR (|)

|	0	1
0	0	1
1	1	1

- One's complement (~)
  - Turns 0 to 1, and 1 to 0
  - E.g., set last three bits to 0
    - x = x & ~7;

Bitwise Operators in C (Continued)

- Shift left (<<)
  - Shift some # of bits to the left, filling the blanks with 0
  - E.g., n << 2 shifts left by 2 bits
    - So, if n is 101_2 (i.e., 5_{10}), then n<<2 is 10100_2 (ie., 20_{10})
  - Multiplication by powers of two on the cheap!
- Shift right (>>)
  - Shift some # of bits to the right
    - For unsigned integer, fill in blanks with 0
    - What about signed integers?
      - Can vary from one machine to another!
  - E.g., n>>2 shifts right by 2 bits
    - So, if n is 10110_2 (i.e., 22_{10}), then n>>2 is 101_2 (ie., 5_{10})
  - Division by powers of two (dropping remainder) on the cheap!

Stupid Programmer Tricks

- Confusing (val % 1024) with (val & 1024)
  - Drops from 1024 bins to ***two*** useful bins
  - You really wanted (val & 1023)
- Speeding up compare
  - For any non-trivial value comparison function
  - Trick: store full hash result in structure

```
struct Nlist *lookup(char *s) {
   struct Nlist *p;
   int val = hash(s); /* no % in hash function */

   for (p = hashtab[val%1024]; p!=NULL; p=p->next)
      if (p->hash == val && strcmp(s, p->key) == 0)
         return p;
   return NULL;
}
```

Summary of Today's Lecture

- Linked lists
  - A list is always the size it needs to be to store its contents
    - Useful when the number of items may change frequently!
  - A list can be rearranged simply by manipulating pointers
    - When items are added/deleted, other items aren't moved
    - Useful when items are large and, hence, expensive to move!
- Hash tables
  - Invaluable for storing (key, value) pairs
  - Very efficient lookups
    - If the hash function is good and the table size is large enough
- Bit-wise operators in C
  - AND (&) and OR (|) – note: they are different from && and ||
  - One's complement (~) to flip all bits
  - Left shift (<<) and right shift (>>) by some number of bits