SOLITON COMPUTATION

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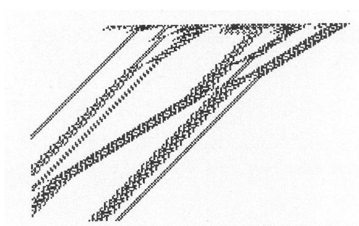
A little history

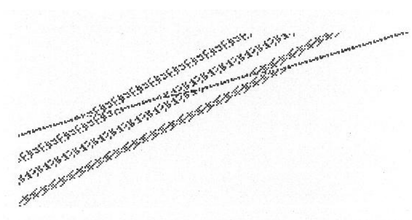
- 1985: solitons in cellular automata
- 1988: embedded addition in 1-d CA
- 1994: particle machines, linear-time arithmetic
- 1996: information transfer in nonintegrable solitons
- 1997: integrable Manakov (vector) solitons
- 1998: state characterization of Manakov solitons
- 2001: universality of gated Manakov solitons
- 2001: experimental information transfer
- 2001: multistable cycles

Parity Rule Filter Automata (PRFA):



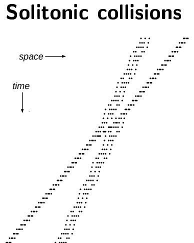
Starting from a random configuration

Solitonic collisions

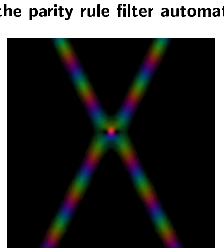


Solitonic collisions, between particles in the parity rule filter automaton

Solitonic collisions



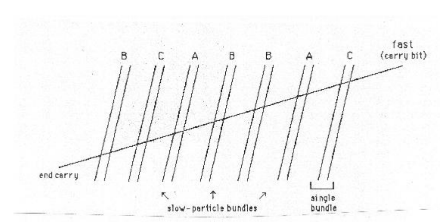
In the parity rule filter automaton



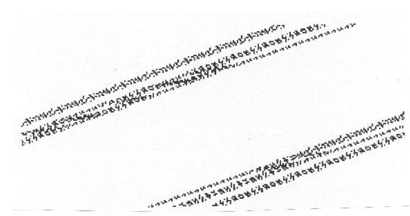
In the nonlinear Schroedinger equation

(Image by Paul Lundquist)

A ripple-carry adder

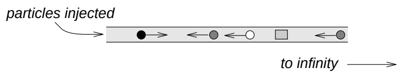


Scheme of a ripple-carry adder embedded in a PRFA



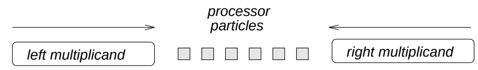
Detail of a typical addition (wrapped)

Particle machines

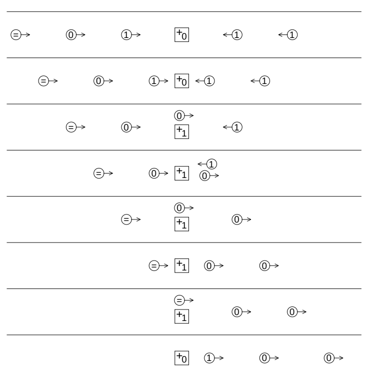


The general picture in one-dimension

Multiplication on a particle machine



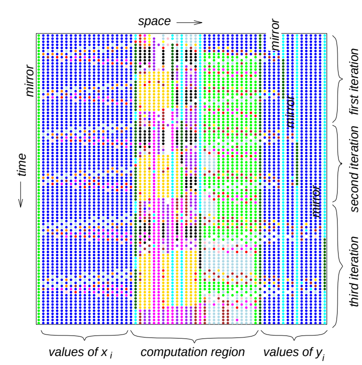
The "soft" systolic array



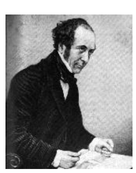
Example of addition

Division (reciprocal)

- Uses Newton iteration a la Leighton; Can do linear-time, arbitrary-precision arithmetic
- Particle machine with 38 types of particles and 79 rules
- A linear, homogeneous, DSP machine



Division in a particle machine

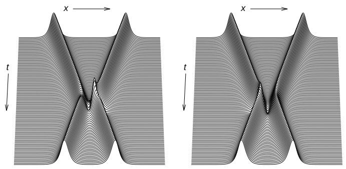


John Scott Russell

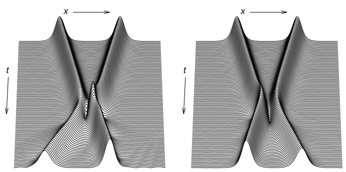
"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion...
I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour...
Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation". — John Scott Russell, Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII.

Soliton on the Scott Russell Aqueduct on the Union Canal near Heriot-Watt University, 12 July 1995 (Photo from Dugald Duncan, Heriot-Watt University, Edinburgh).

Information transfer in collisions



Collision in the (integrable) nonlinear Schroedinger equation; no information is transfered. Relative phases at left and right differ.



Collision in the saturable (nonintegrable) Schroedinger equation; information is transfered, but at the expense of radiation. Relative phases at left and right differ.

A surprise

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Inelastic collision and switching of coupled bright solitons in optical fibers

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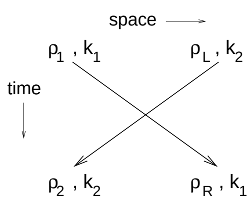
$$iq_{1x}+q_{1tt}+2\mu(|q_1|^2+|q_2|^2)q_1=0,$$

$$iq_{2x}+q_{2tt}+2\mu(|q_1|^2+|q_2|^2)q_2=0,$$

$$q_1=\frac{\alpha_1 e^{\eta_1}+\alpha_2 e^{\eta_2}+e^{\eta_1+\eta_1^*+\eta_2+\delta_1}+e^{\eta_1+\eta_2+\eta_2^*+\delta_2}}{1+e^{\eta_1+\eta_1^*+R_1}+e^{\eta_1+\eta_2^*+\delta_0}+e^{\eta_1^*+\eta_2+\delta_0^*}+e^{\eta_2+\eta_2^*+R_2}+e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+R_3}},$$

$$q_2=\frac{\beta_1 e^{\eta_1}+\beta_2 e^{\eta_2}+e^{\eta_1+\eta_1^*+\eta_2+\delta_1'}+e^{\eta_1+\eta_2+\eta_2^*+\delta_2'}}{1+e^{\eta_1+\eta_1^*+R_1}+e^{\eta_1+\eta_2^*+\delta_0}+e^{\eta_1^*+\eta_2+\delta_0^*}+e^{\eta_2+\eta_2^*+R_2}+e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+R_3}},$$

State transformations

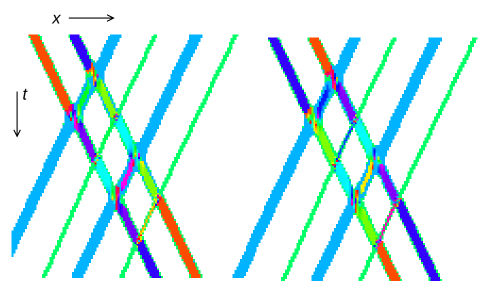


$$\rho=q_1(x,t)/q_2(x,t)$$

$$\rho_2=\frac{[(1-g)/\rho_1^*+\rho_1]\rho_L+g\rho_1/\rho_1^*}{g\rho_L+(1-g)\rho_1+1/\rho_1^*},$$

$$g(k_1,k_2)=\frac{k_1+k_1^*}{k_2+k_1^*}.$$

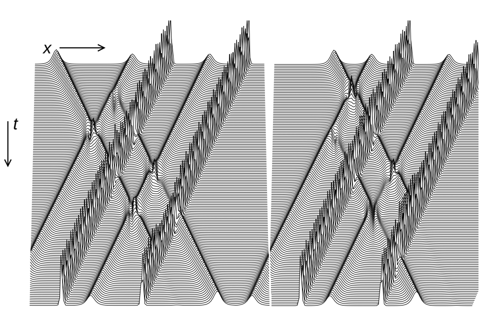
Phase switching gate



0 --> 1 *1 --> 0*

Phase-switching NOT gate

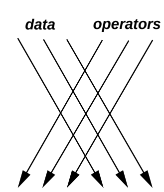
Energy switching gate, a kind of dual

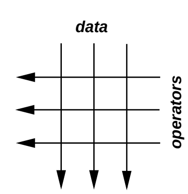


0 --> 1 *1 --> 0*

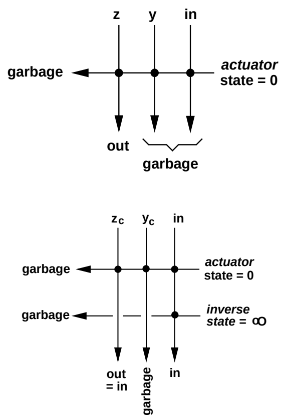
Energy-switching NOT gate

Building a computer





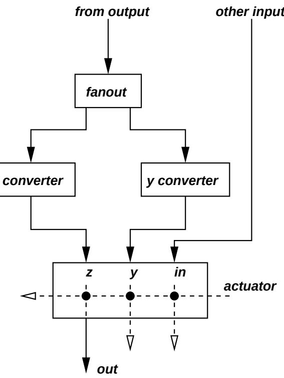
COPY and FANOUT



wire crossings later!

... plus NOT, ONE gates in a similar fashion

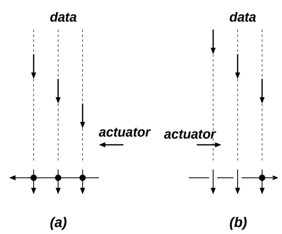
NAND



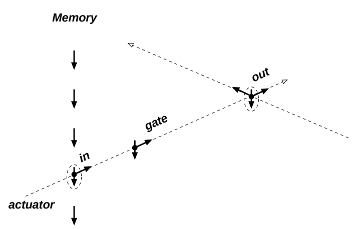
z converter: 0/1 yields z's for a ONE/NOT gate

y converter: 0/1 yields y's for a ONE/NOT gate

Wiring

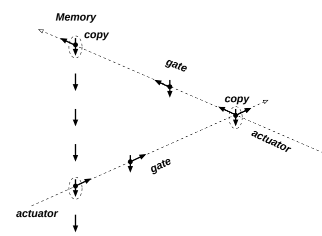


crossing wires

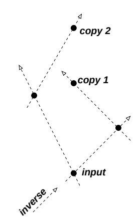


moving frame

Need for two speeds

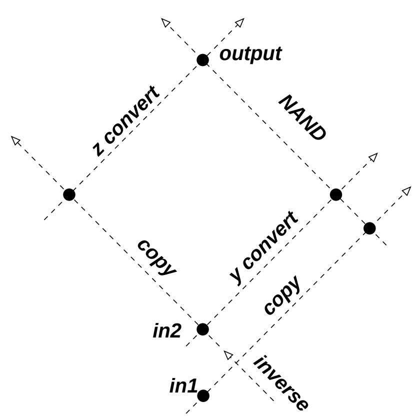


typical operation



true fanout with two speeds

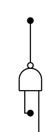
NAND



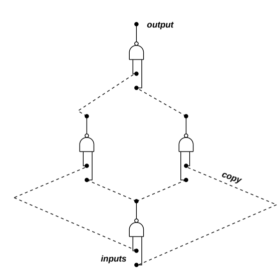
Complete NAND

A second speed is needed to use the output

General logic

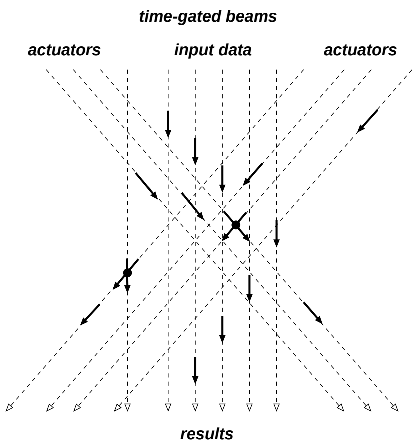


Conventional representation of a NAND



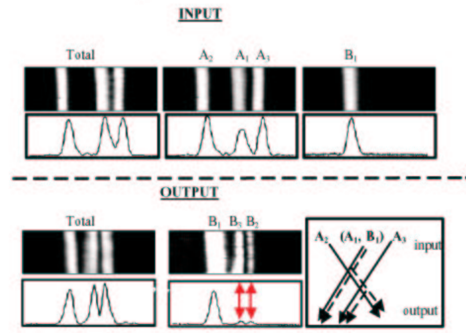
Implementing an XOR with NANDs

The big picture: spatial solitons in a photorefractive crystal

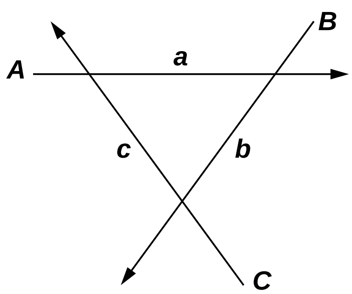


Experimental results

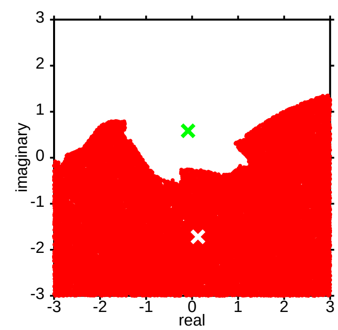
Anastassiou, Fleischer, Carmon, Segev, Steiglitz, submitted to Optics Letters



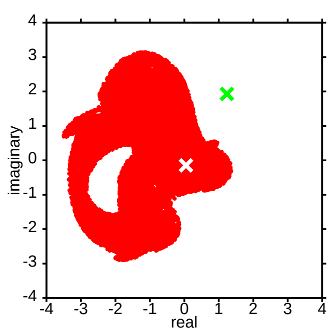
Multistability: all-optical set-reset flipflops



Basins of attraction



Basins of attraction, another example



Basins of attraction, four beams