Physical Limits to Communication

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The limits posed by physics to the quantity of information that can be transmitted with a certain amount of power are investigated. The same ultimate limits are found for transmission of information encoded using matter and massless fields.

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How much information can two parties exchange through a communication channel given certain energy resources? On one hand, quantum mechanics constrains the physical resources of any system in its capacity to store, transmit, and process information [1–4]. On the other hand, general relativity constrains the amount of information that can be stored in a finite volume of space through black hole thermodynamics [5–10]. Here we study the fundamental limits to the transmission of information by joining these two approaches: we provide some arguments which indicate that the ultimate communication rate might be obtained by transmitting information encoded into objects on the verge of becoming black holes. The appeal of our arguments is that they stem from simple general considerations on information and entropy in physical systems, which make use of basic concepts of relativity, quantum mechanics, and information theory. On the same lines, a different approach was recently given in Ref. [7]: there black hole thermodynamics is conjugated with quantum information theory to obtain bounds on the capacity of channels which are consistent with the ones we find here.

It is known that no energy need be dissipated in order to communicate [11]. However, since information must be encoded in some physical medium, communication is a dynamical process and always requires a certain amount of energy to be invested, even in the ideal case of noiseless transmission. By increasing this energy it is possible to obtain higher transfer rates since faster dynamics are achievable and larger spaces of codewords become available: the purpose of this Letter is to analyze this energy-rate connection. In our analysis we describe a communication channel simply as a region of space with transverse cross section A that connects the sender Alice and the receiver Bob, who are supposed to share the same inertial reference frame (see Fig. 1). Assuming that a power P is employed in the communication task, we show that the maximum rate R (i.e., the number of bits or qubits transmitted per unit time) is limited by

$$R \lesssim \sqrt{\frac{AP}{l_p^2 \hbar}}, \tag{1}$$

where $l_p = \sqrt{\hbar G/c^3} \simeq 1.6 \times 10^{-35}$ m is the Planck length (G being the gravitational constant). As is customary in information theory, we leave out of the energy balance the energy needed by Alice to prepare the signal and by Bob to decode it.

Channel capacity bound.—Consider the scenario of Fig. 1: Alice encodes the information into slabs of material with rest-mass density ρ and sends them to Bob at a speed v. The slabs can be thought of as "pages" written so densely that *all* the available degrees of freedom are employed to store information. From the point of view of Alice and Bob, each slab of rest length L will be Lorentz contracted by a factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ in the longitudinal direction. If Alice fills the channel continuously, the time between the arrival of two successive slabs

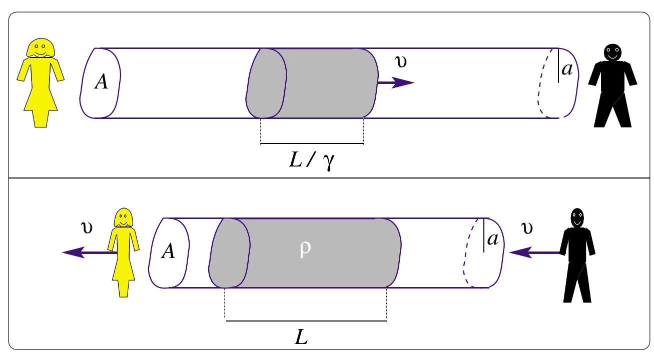

FIG. 1 (color online). Transmission line for massive objects. Alice continuously sends Bob information encoded into slabs of material of cross section A (with largest transverse dimension a) and rest length L at a speed v. The upper graph depicts the situation in Alice and Bob's reference frame where the slab is Lorentz contracted by a factor γ; the lower graph depicts the situation in the slab frame where Alice and Bob are Lorentz contracted by the same amount.

is $\tau = L/(\gamma v)$. The average power transmitted is then

$$P = \frac{E}{\tau} = \frac{mc^2\gamma^2 v}{L}, \tag{2}$$

where $m = AL\rho$ is the rest mass of the slabs she sends and $E = mc^2\gamma$ is their energy as seen in Alice and Bob's reference frame. The information transmission rate is given by

$$R = I/\tau, \tag{3}$$

where I is the information (i.e., the entropy for the noiseless case we are considering) contained in the slab. To estimate the maximum possible I, we employ the Bekenstein bound [7,9,10], which limits the information $\mathcal{I}$ that can be stored in a spherical volume of space with radius r on an object with rest energy $\mathcal{E}$. It states that

$$\mathcal{I} \leq \frac{2\pi}{\ln 2}\frac{\mathcal{E}r}{c\hbar}. \tag{4}$$

The Bekenstein bound, although generally accepted, has not been derived from basic physical postulates and does not take into account possible quantum gravity effects. It assumes that the number of physical fields is limited and it assumes linearity, i.e., that there is no interaction between different field species [4,10]. (For further discussions on the validity of the Bekenstein bound we refer the reader to Refs. [7,12].) However, we can still use this bound as long as Alice employs a finite number of types of information carriers (presently only very few different physical species are known) and if she distributes them homogeneously in the channel so that in average the interactions cancel: namely, we consider a mean-field approach redefining the normal excitations of the channel as the effective information carriers. In this respect, the information flows by means of an effective medium made of independent particles [13]. Clearly this last argument cannot be applied to the gravitational field, so that, as is clarified in the following, our treatment holds only for weakly self-gravitating objects. Moreover, it cannot be applied also to disordered systems, where the motion is diffusive rather than ballistic; however, it would be quite surprising if disorder would allow a more efficient way of information transfer [13].

For the sake of definiteness, we consider a channel with circular cross section. Different geometries just introduce numerical factors that do not change the scaling properties of the capacity. In order to apply the Bekenstein bound to our channel, as shown in Fig. 2, we enclose the channel into a collection of overlapping spheres of radius $r = \sqrt{a^2 + l^2}$, where a is the radius of the channel and $2l$ is the distance between successive spheres (l is a free parameter, which is fixed later to derive the tightest possible bound). This procedure is necessary since the Bekenstein bound applies to spherical volumes. Moreover, chaining the successive spheres together is justified since the spheres are weakly interacting: they can be considered independent even if partially overlapping. Equation (4) implies that the amount of information δI contained in each sphere is upper bounded by

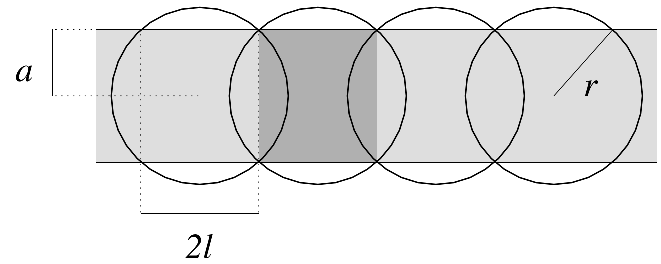

FIG. 2. In order to apply the Bekenstein bound (4) we enclose the channel with overlapping spheres of radius r which are separated by a distance l (as measured in the rest frame of the information carriers).

$$\delta I \leq \frac{2\pi}{\ln 2}\frac{\delta mc\sqrt{a^2 + l^2}}{\hbar}, \tag{5}$$

where $\delta m = \frac{4}{3}\pi\rho[(a^2 + l^2)^{3/2} - l^3]$ is the rest mass contained in the sphere. Equation (5) implies that there is an amount of information of $4\pi^2/\ln 2$ bits for each Compton wavelength $2\pi\hbar/(\delta mc)$ that can fit into the radius r. The spherical covering of the channel of Fig. 2 is overestimating its volume, since it entirely contains the channel and the parts in which the spheres overlap are counted twice. Hence, an upper bound for the information contained in a slab of length L can be obtained by summing over the $L/(2l)$ spheres that contain the slab, i.e.,

$$I \leq \frac{L}{2l}\delta I. \tag{6}$$

Replacing Eqs. (6) and (2) into Eq. (3), we find the rate in terms of the power P as

$$R \leq \frac{2\pi f(l/a)}{3\ln 2} a^2 \sqrt{\frac{\rho P v}{\pi\hbar^2}}, \tag{7}$$

where $f(x) \equiv \sqrt{1 + x^2}[(1 + x^2)^{3/2} - x^3]/x$ is a dimensionless quantity that derives from the geometry of the spherical covering. Since this equation applies for any values of the parameter l, in order to get the most tight upper bound we should choose x that minimizes $f(x)$, i.e., $x_0 \simeq 0.82$ for which $f(x_0) \simeq 2.54$.

It would seem that, even with fixed power P, the upper bound of Eq. (7) could be arbitrarily increased by increasing the mass density ρ of the slabs. However, a limit on ρ is given by the fact that too massive slabs eventually collapse to black holes [14] and that our analysis does not apply in this regime. Whether or not black holes are a suitable medium for transmitting information depends on currently unknown features of quantum gravity, in particular, whether black hole evaporation preserves or destroys information [5–7]. If it preserves information, i.e., if black hole evaporation is a unitary process, the black holes can indeed be used for communication. (Of course, Bob may have some serious decoding to do in order to decipher Alice's message out of the Hawking radiation.) If, by contrast, black hole evaporation destroys information, this is not a suitable medium for communication. In this case, however, it is still possible to approach arbitrarily the above bounds by using systems poised on the edge of gravitational collapse. This "black hole communication" limit corresponds to the fact that the Bekenstein bound is achievable using black holes [9]. A rigorous treatment of the gravitational effects of the information carriers would require us to consider the entire geometry of the communication line. However, to prevent gravitational collapse the simplest requirement we can impose is that any sphere of radius r_0 contains a rest mass smaller than the Schwarzschild mass $c^2 r_0/(2G)$, [14]. This is enough to provide the correct scaling properties of the capacity bound. For our channel, the Schwarzschild condition translates to

$$\rho \le \frac{c^2}{4\pi G a^2}, \tag{8}$$

which limits the rate as

$$R \le \frac{2f(x_0)}{3\ln 2}\frac{a}{l_p}\sqrt{\frac{v}{c}\frac{P}{\hbar}} \le \frac{2f(x_0)}{3\ln 2}\frac{a}{l_p}\sqrt{\frac{P}{\hbar}}. \tag{9}$$

This reduces to Eq. (1) since $A \sim a^2$ and the numerical coefficients are of order 1. Interestingly, an analogous $\sqrt{P}$ scaling was obtained through a black hole thermodynamics argument in Ref. [7], where it was shown to apply only in a "low-power" regime. For our model this corresponds to considering $P < c^2\hbar/a^2$ which, according to Eq. (1), represents a transmission rate lower than 1 bit per Plank time $t_p \equiv l_p/c$. Whereas our analysis does not take into account the thermodynamics of black holes, the analysis of [7] does not contemplate the possibility that the information carriers themselves can collapse into black holes. In this respect, it is possible that a better understanding of the communication bound can be achieved by joining the two approaches.

Notice that our bound (1) depends explicitly on all three fundamental constants c, G, and h, whereas the Bekenstein bound depends only on c and h. This is not surprising since the information rate has the dimension of time^{-1}: if the rate is to be expressed in natural units [15] then by necessity G will be introduced from the Planck time t_p dependence on all three constants.

Discussion.—Is the capacity of Eq. (1) achievable? One needs to saturate the Bekenstein bound (5), the Schwarzschild condition on the mass (8), and use high speed $v \to c$. This corresponds to a highly exotic communication channel where Alice encodes information into all the available degrees of freedom of a sequence of Lorentz-contracted black holes with Schwarzschild radius equal to the channel width a. Clearly, in this regime, gravitational effects come into play that might impede the possibility of achieving the capacity (1).

In deriving the bound, we have used a communication protocol where massive objects of density ρ are exchanged at a speed v. However, both these parameters enter in the final expression (7) only through the power P. This is a clue that such a bound may be applied also to other regimes, such as the case in which the information is encoded into massless objects, e.g., electromagnetic waves. In fact, since both the Bekenstein bound (4) and the Schwarzschild condition (8) can be extended to all known quantum fields [10] it is reasonable to assume that Eq. (1) is indeed a general statement on the maximum communication rate allowed by nature. Clearly a rigorous derivation of this bound would require a quantum theory of gravity which is still beyond our grasp.

A support to our intuition about Eq. (1) derives from considering the case of photonic communication channels, where Alice is employing photons to encode information. For the sake of simplicity, analyze the case of a long communication line, where the transverse dimension a is much smaller than the channel length L. Here Alice can use only longitudinally propagating modes of the field (all the others modes eventually exit the channel and do not reach Bob). The maximum rate she can achieve is given by [2,3,13]

$$R \le \frac{1}{\ln 2}\sqrt{gN}\sqrt{\frac{\pi P}{3\hbar}}, \tag{10}$$

where g is the number of orthogonal polarizations of the field and N is the number of parallel longitudinal modes that are employed in the communication. The product gN counts the number of parallel channels employed, since it is the number of energy-degenerate information carriers that reach the receiver. In deriving Eq. (10), no general relativistic effects were taken into account: this means that such a bound might fail if one considers photons with a wavelength smaller than the Planck length, where gravitational effects come into play. Nonetheless, joining the Bekenstein bound (4) with the Schwarzschild condition (8) one can give an upper bound to the number of modes N (i.e., parallel channels) that can fit into the channel cross section A, as

$$gN < \frac{A}{4l_p^2}, \tag{11}$$

which essentially implies that the best Alice can do to transmit information is to encode it into photons of transverse wavelength of the order of the Planck length l_p (i.e., on the verge of becoming "photonic black holes"). Notice that Eq. (11) can be interpreted as an instance of the Susskind holographic bound [7,16]. In this respect, it limits the number of polarizations and/or bosonic species g that are allowed by nature, as also assumed in the Bekenstein bound derivation [10]. Replacing this relation into Eq. (10) we reobtain the bound (1), which hence applies also to photonic communication lines.

An important consequence of the bound (1) is the scaling law of the rate in terms of the number N of parallel channels used. At least for the optimum case, the transmission rate for communicating with N parallel channels always scales as $\sqrt{N}\sqrt{P/\hbar}$. One of the assumptions of our calculations was the absence of interactions among the information carriers. It was pointed out in Ref. [17] that interactions among fields can be used to increase the entropy and hence the communication rate. However, this does not necessarily entail that the Bekenstein bound can be beaten since one would have to consider also the energy introduced by the physical mediators of the interactions [18]. Interactions might help overcome the $\sqrt{N}$ bound only in those communications scenarios where some degrees of freedom of the available resources are not used for information transmission but are used instead to set up a suitable communication line [19].

As a final remark, we notice that in the case in which Alice and Bob are equipped with some prior entanglement, the bit rate can be increased, but only by a factor of 2 [1,20] which does not change the substance of our bound.

Conclusions.—Starting from a simple communication model, we have given the bounds that physics imposes on the communication rate for a channel of cross section A to which a power P is devoted. We presented separate arguments, leading to the same result, for communication through the exchange of matter and radiation. A scaling law proportional to the square root of the number of channels was found for communicating in the most extreme conditions allowed by nature.

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