Implicit Surfaces

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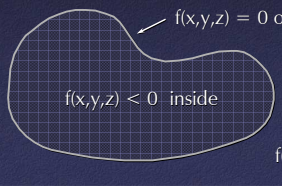
COS 526, Fall 2006

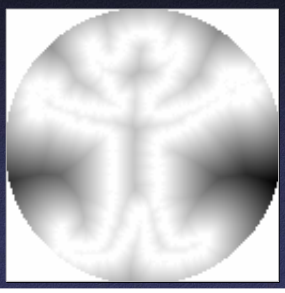
Implicit Surfaces

- Surface defined implicitly by function:
  - $f(x, y, z) < 0$ (inside)
  - $f(x, y, z) = 0$ (on surface)
  - $f(x, y, z) > 0$ (outside)



Implicit Surfaces

- Surface = level set of volumetric function



Kazhdan

Implicit Surfaces

- Normals defined by partial derivatives
  - $\text{Normal}(x, y, z) = (\partial f/\partial x, \partial f/\partial y, \partial f/\partial z)$

Normals Tangents Curvatures

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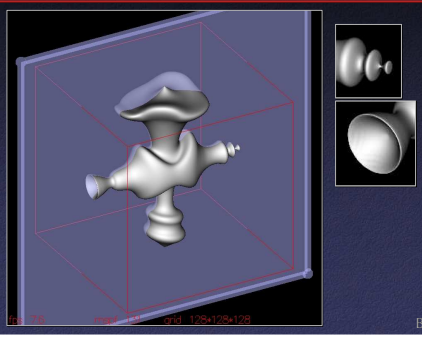
Implicit Surfaces



Bourke

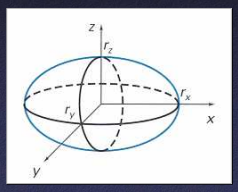
Common Implicit Functions

- Euclidean distance transform
  - "Unsigned distance"
  - Touches, but does not cross, zero: can encounter numerical problems
- Signed distance function

Implicit Surface Properties

- Efficient check for whether point is inside
  - Evaluate $f(x, y, z)$ to see if point is inside/outside/on

$$\left(\frac{x}{r_x}\right)^2+\left(\frac{y}{r_y}\right)^2+\left(\frac{z}{r_z}\right)^2-1=0$$



H&B Figure 10.10

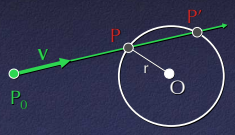
Implicit Surface Properties

- Efficient surface intersections
  - Substitute to find intersections

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

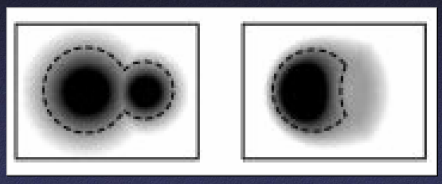
Substituting for P, we get:
$|P_0 + tV - O|^2 - r^2 = 0$

Solve quadratic equation:
$at^2 + bt + c = 0$
where:
$a = 1$
$b = 2\,V \bullet (P_0 - O)$
$c = |P_0 - C|^2 - r^2 = 0$



Implicit Surface Properties

- Efficient boolean operations (CSG)
  - Union, difference, intersection



Union Difference

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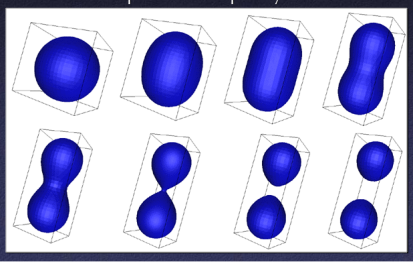
Implicit Surface Properties

- Efficient topology changes
  - Surface not represented explicitly



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Implicit Surface Properties

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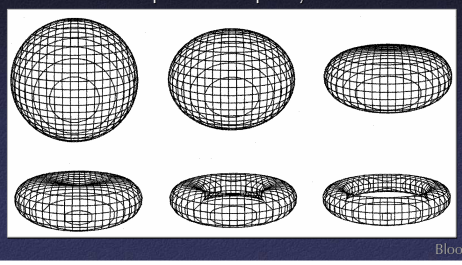

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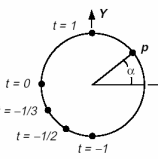
Comparison to Parametric Surfaces

- Implicit: Efficient intersections & topology changes
- Parametric: Efficient "marching" along surface & rendering



equiangular parametric (transcendental trigonometric)
$\boldsymbol{p} = (\cos(\alpha), \sin(\alpha)),\ \alpha \in [0, 2\pi]$

non-equiangular parametric (rational)
$\boldsymbol{p} = (\pm(1-t^2)/(1+t^2),\ 2t/(1+t^2)),\ t \in [-1, 1]$

implicit
$\boldsymbol{p}_x^2+\boldsymbol{p}_y^2-1 = 0$

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Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - Blobby models
  - Skeletons
  - Procedural
  - Samples
  - Variational

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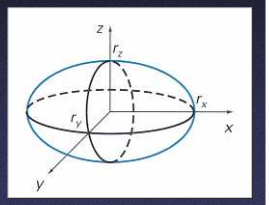
Implicit Algebraic Surfaces

- Implicit function is polynomial
  - $f(x,y,z)=ax^d+by^d+cz^d+ex^{d-1}y+fx^{d-1}z+gy^{d-1}x+\ldots$

$$\left(\frac{x}{r_x}\right)^2+\left(\frac{y}{r_y}\right)^2+\left(\frac{z}{r_z}\right)^2-1=0$$



H&B Figure 10.10

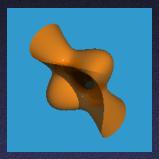
Implicit Algebraic Surfaces

- Most common form: quadrics
  - $f(x,y,z)=ax^2+by^2+cz^2+2dxy+2eyz+2fxz+2gx+2hy+2jz+k$
  - Sphere
  - Ellipsoid
  - Paraboloid
  - Hyperboloid

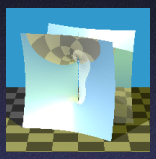
Menon

Implicit Algebraic Surfaces

- Higher degree algebraics



Cubic



Quartic

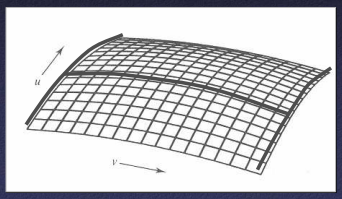
Degree six

Implicit Algebraic Surfaces

- Function extends to infinity
  - Must trim to get desired patch

Implicit Algebraic Surfaces

- Equivalent to a parametric surface
  - Tensor product patch of degree *m* and *n* curves yields algebraic implicit with degree 2*mn*



Bicubic parametric equivalent to implicit of degree 18

Implicit Algebraic Surfaces

- Intersection
  - Intersection of degree m and n algebraic surfaces yields curve with degree mn

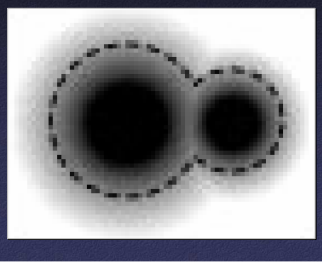
Intersection of bicubic patches has degree 324…

Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - Blobby models
  - Skeletons
  - Procedural
  - Samples
  - Variational

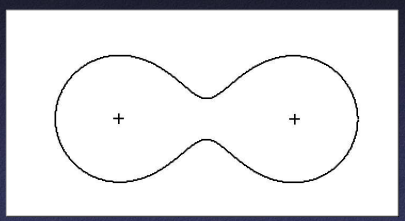
Blobby Models

- Implicit = sum of spherical basis functions



Blobby Models

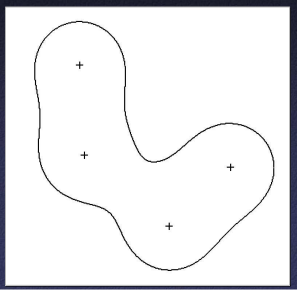
- Sum of two blobs



Turk

Blobby Models

- Sum of four blobs



Turk

Blobby Models

- Blobby molecules

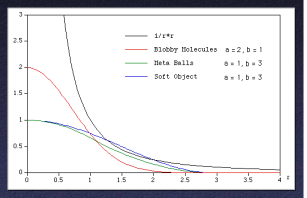
$$D(r) = ae^{-br^2}$$

- Meta balls

$$D(r) = \begin{cases} a\left(1-\frac{3r^2}{b^2}\right) & 0 \le r \le b/3 \\ \frac{3a}{2}\left(1-\frac{r}{b}\right)^2 & b/3 \le r \le b \\ 0 & b \le r \end{cases}$$

- Soft objects

$$D(r) = \begin{cases} a\left(1-\frac{4r^6}{9b^6}+\frac{17r^4}{9b^4}-\frac{22r^2}{9b^2}\right) & r \le b \\ 0 & r \ge b \end{cases}$$



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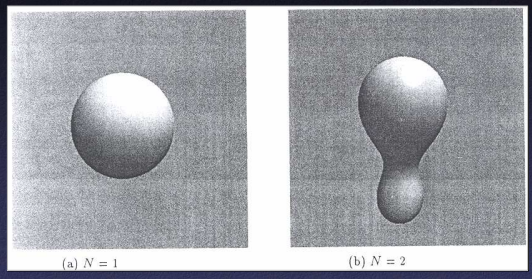
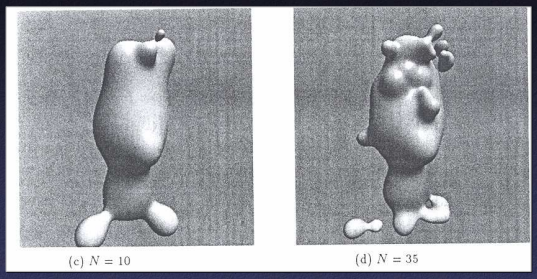
Blobby Model of Face



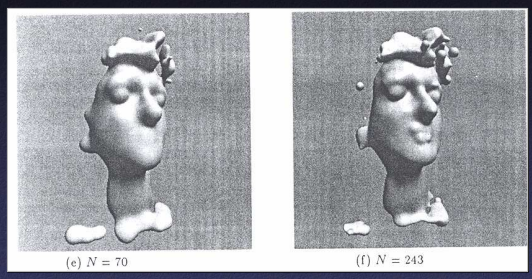
(a) $N = 1$ (b) $N = 2$

Blobby Model of Face



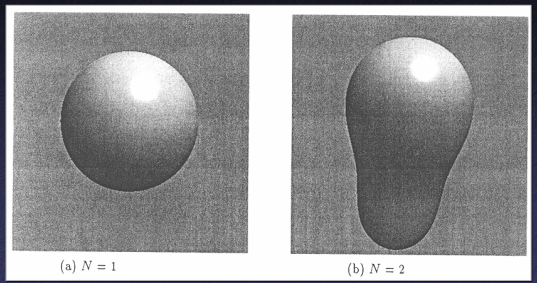
(c) $N = 10$ (d) $N = 35$

Blobby Model of Face



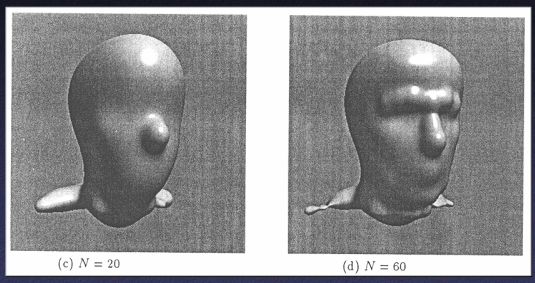
(e) $N = 70$ (f) $N = 243$

Blobby Model of Head



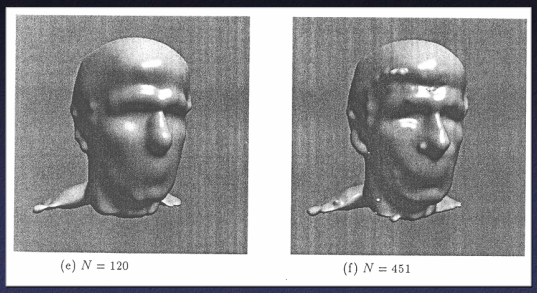
(a) $N = 1$ (b) $N = 2$

Blobby Model of Head



(c) $N = 20$ (d) $N = 60$

Blobby Model of Head



(e) $N = 120$ (f) $N = 451$

Blobby Models

Objects resulting from CSG of implicit soft objects and other primitives

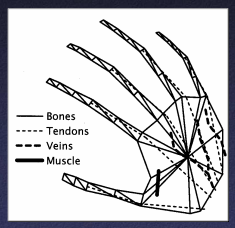
Menon

Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - Blobby models
  - Skeletons
  - Procedural
  - Samples
  - Variational

Skeletons

- Convolution surfaces



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Implicit Surface Representations

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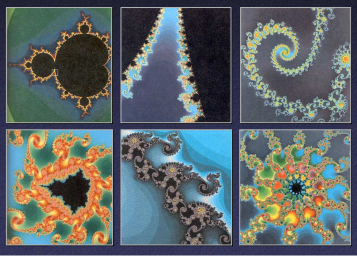
Procedural Implicits

- Function is result or limit of procedure
  - Example: Mandelbrot set



H&B

Implicit Surface Representations

- How do we define implicit function?
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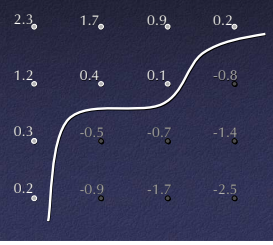
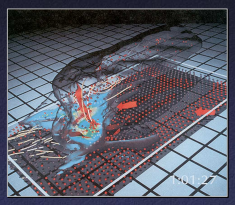
Sampled Functions

- Most common example: voxels
  - Interpolate samples stored on regular grid



Sampled Functions

- Acquired from simulations or scans

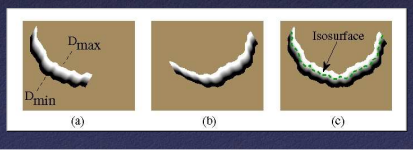


Airflow Inside a Thunderstorm
(Bob Wilhelmson, University of Illinois at Urbana-Champaign)

Visible Human
(National Library of Medicine)

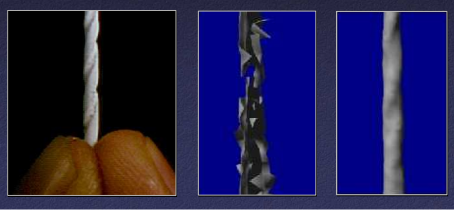
Volumetric Reconstruction

- Idea: use implicit functions to merge models (especially scans from 3D rangefinder)
  - Generate implicit function for each scan
  - Compute average (possibly weighted)
  - Extract isosurface



Volumetric Reconstruction Benefits

- Always generates a manifold surface
- Can control sampling density
- Averages surfaces – helps eliminate noise



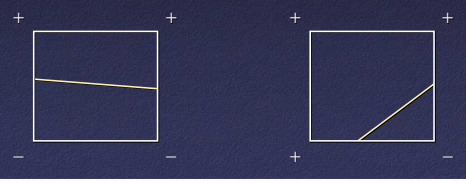
Curless

Volumetric Reconstruction Drawbacks

- Represent a 3D entity rather than 2D
  - Running time
  - Storage
- Resampling step – bandlimits the function
- Generates consistent topology, but not always the topology you wanted
- Problems with very thin surfaces

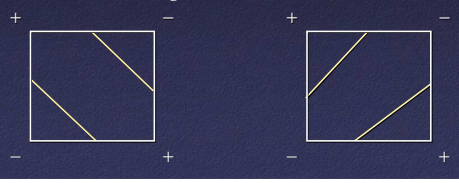
Marching Cubes

- Lorensen & Cline, 1987
- Consider 2D analogy: "marching squares"
- Look at signs at corners of square



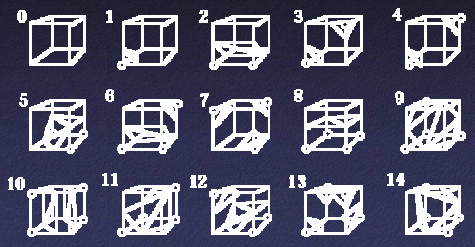
Marching Cubes

- Signs of corners → lookup table → polygons
- Actual values at corners: locations of vertices along edges of square / cube
- Sometimes ambiguous



Marching Cubes

- Same idea can be scaled up to 3D



Implicit Surface Representations

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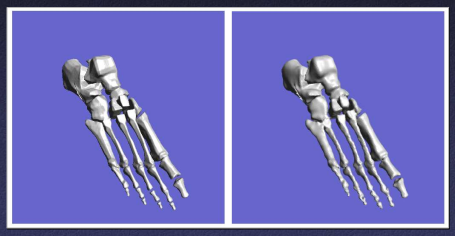
Variational Implicit Surfaces

- Convert differential equation on surface to differential equation on signed distance function



Implicit Function Evolution

- Let φ = implicit function, s = surface
- Then

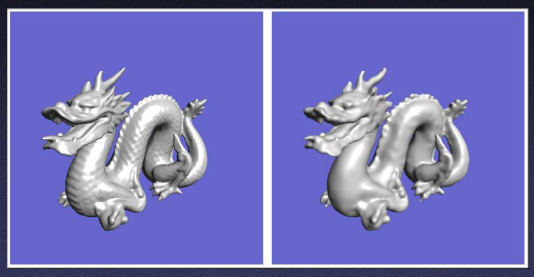
$$s(t) = \{x : \varphi(x,t) = 0\}$$

$$\frac{d\varphi(s(t),t)}{dt} = \frac{\partial \varphi}{\partial t} + \nabla\varphi \cdot \frac{ds}{dt} = 0$$

$$\frac{\partial \varphi}{\partial t} = -\nabla\varphi \cdot \frac{ds}{dt}$$

- Relates motion of surface to change in φ

Variational Implicit Surfaces



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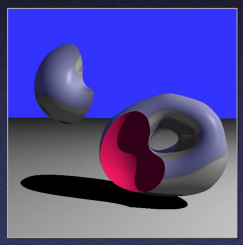
Implicit Surface Summary

- Advantages:
  - Easy to test if point is on surface
  - Easy to compute intersections/unions/differences
  - Easy to handle topological changes
- Disadvantages:
  - Indirect specification of surface
  - Hard to describe sharp features
  - Hard to enumerate points on surface
    - Slow rendering

Rendering Implicit Surfaces

- How do we render images of implicit surfaces?
  - Polygonization
  - Ray tracing
  - Contours
  - Floating particles



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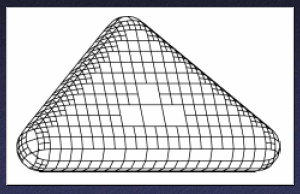
Rendering with Polygons

- Different polygonization strategies



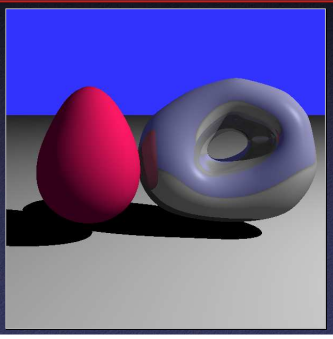
Adaptive Polygonalization

Marching Cubes

Bloomenthal

Lorensen

Rendering with Ray Tracing



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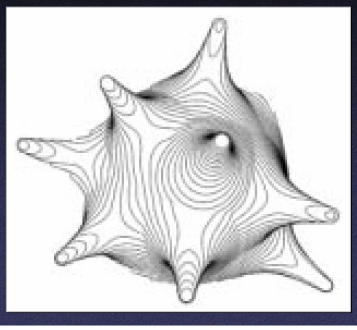
Rendering with Contours



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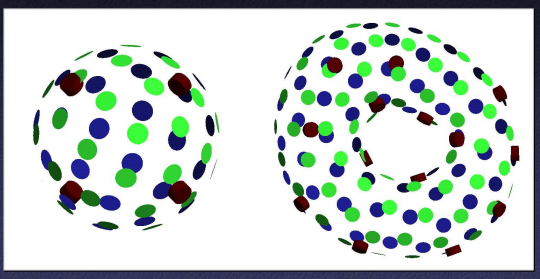
Rendering with Floating Particles



Turk

Summary

Feature	Polygonal Mesh	Implicit Surface	Parametric Surface	Subdivision Surface
Accurate	No	Yes	Yes	Yes
Concise	No	Yes	Yes	Yes
Intuitive specification	No	No	Yes	No
Local support	Yes	No	Yes	Yes
Affine invariant	Yes	Yes	Yes	Yes
Arbitrary topology	Yes	Yes	No	Yes
Guaranteed continuity	No	Yes	Yes	Yes
Natural parameterization	No	No	Yes	No
Efficient display	Yes	No	Yes	Yes
Efficient intersections	No	Yes	No	No