


## Shape Representations for Retrieval and Matching

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### 3D Representations

What properties are required for analysis and retrieval?

Property	Editing	Display	Analysis	Retrieval
Intuitive specification	Yes	No	No	No
Guaranteed continuity	Yes	No	No	No
Guaranteed validity	Yes	No	No	No
Efficient boolean operations	Yes	No	No	No
Efficient rendering	Yes	Yes	No	No
Accurate	Yes	Yes	?	?
Concise	?	?	?	Yes

### 3D Representations for Retrieval

Some desirable properties

- Quick to compute
- Efficient to match
- Concise to store
- Invariant to transforms
- Insensitive to noise
- Insensitive to topology
- Robust to degeneracies
- Discriminating

### 3D Representations for Retrieval

Statistical shape descriptors

- Voxels, moments, wavelets, ...
- Attributes, histograms, ...

Structural shape descriptors

- Feature-based methods
- Part-based methods
- Skeletons

### Statistical Shape Descriptors

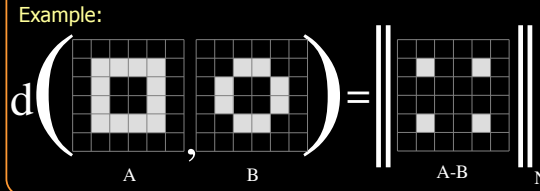
<p>Alignment-dependent</p> <ul style="list-style-type: none"> <li>• Voxels</li> <li>• Wavelets</li> <li>• Moments</li> <li>• Extended Gaussian Image</li> <li>• Spherical Extent Function</li> <li>• Spherical Attribute Image</li> </ul>	<p>Alignment-independent</p> <ul style="list-style-type: none"> <li>• Shape distributions</li> <li>• Shape histograms</li> <li>• Harmonic descriptor</li> </ul>
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### Voxels

Use voxel values as feature vector (shape descriptor)

- Feature space has  $N^3$  dimensions (one dimension for each voxel)
- $d(A,B) = \|A-B\|_N$

Example:




### Voxels


Image courtesy of Misha Kazhdan

Can store distance transform (DT) in voxels

- $\|A-DT(B)\|_1$  represents sum of distances from every point on surface of A to closest point on surface of B



Surface



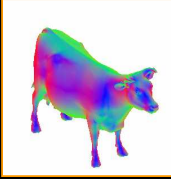
Distance Transform

### Voxels


Image courtesy of Misha Kazhdan

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Surface



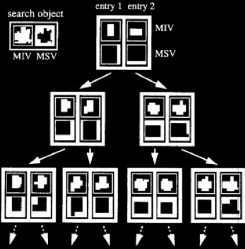
Distance Transform

### Voxels

Image courtesy of Daniel Keim, SIGMOD 1999









Can build hierarchical search structure

- e.g., interior nodes store MIV and MSV



### Voxel Retrieval Experiment

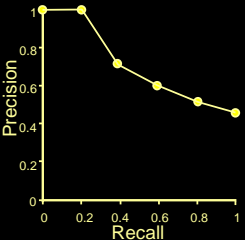
Test database is Viewpoint household collection  
1,890 models, 85 classes

 153 dining chairs	 25 livingroom chairs	 16 beds	 12 dining tables
 8 chests	 28 bottles	 39 vases	 36 end tables

### Evaluation Metric

Precision-recall curves


- Precision = retrieved\_in\_class / total\_retrieved
- Recall = retrieved\_in\_class / total\_in\_class












### Evaluation Metric

Precision-recall curves

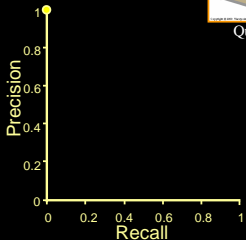
- Precision = 0 / 0
- Recall = 0 / 5

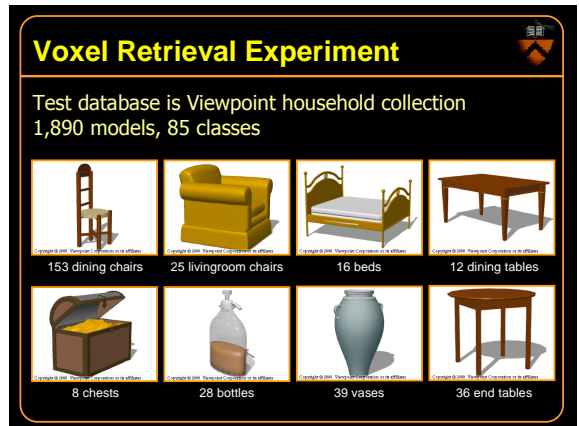
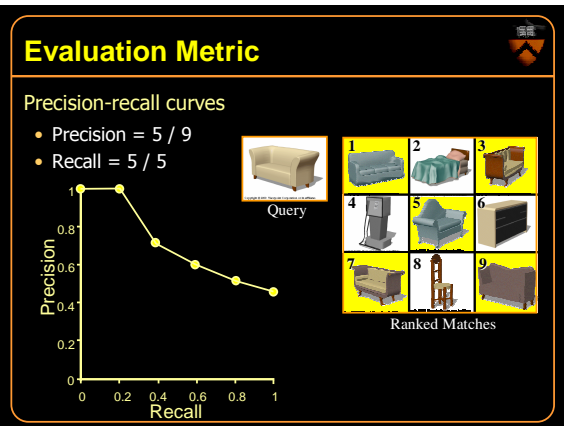
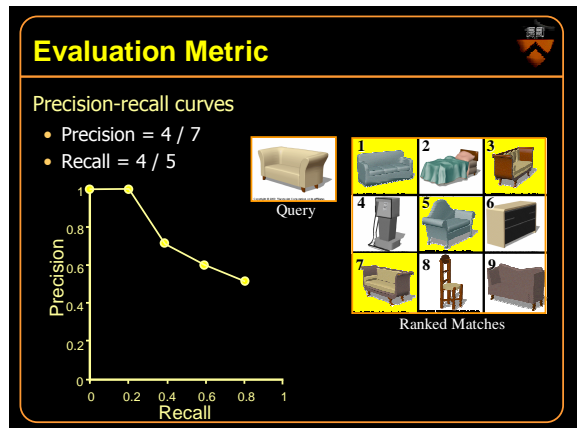
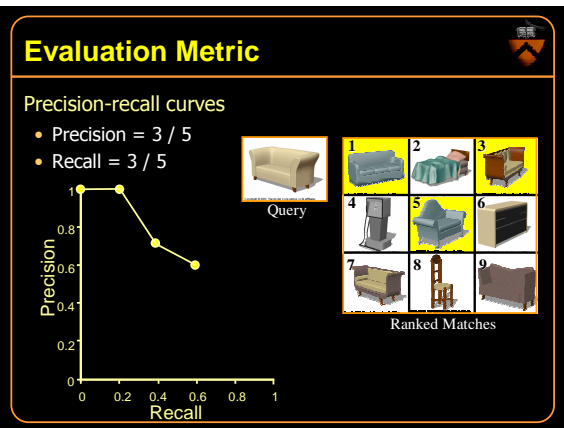
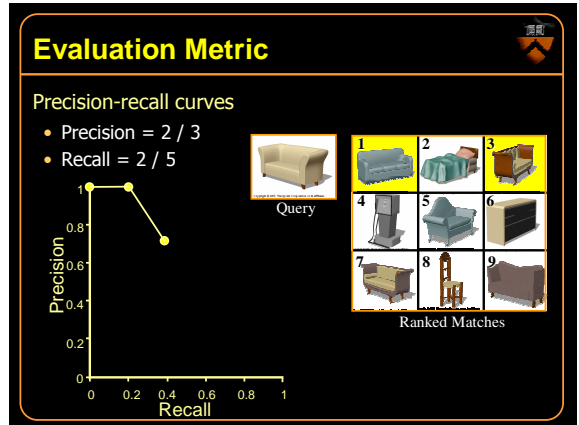
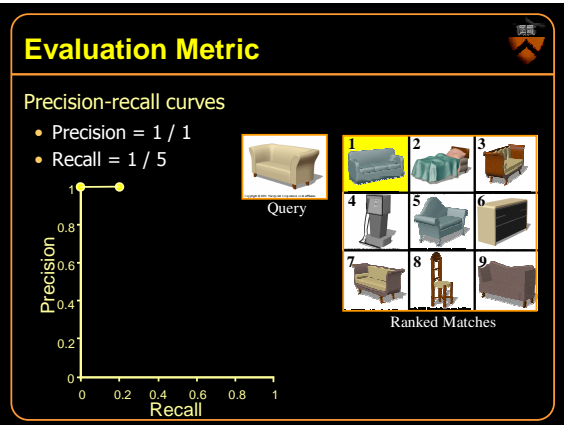


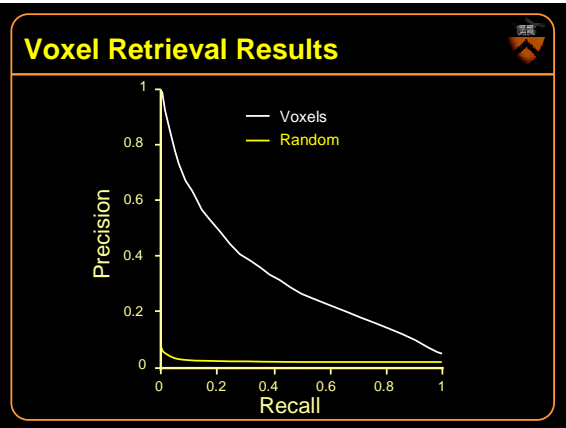
Query

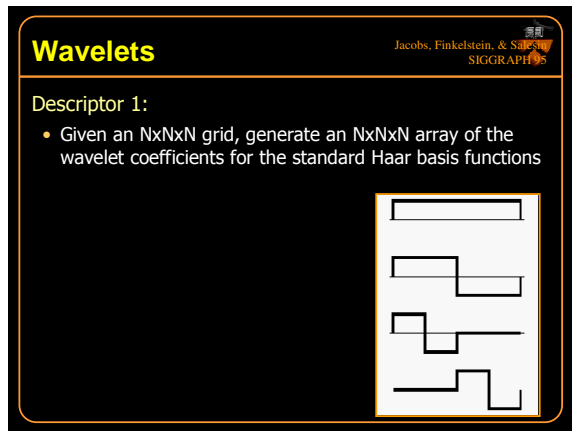
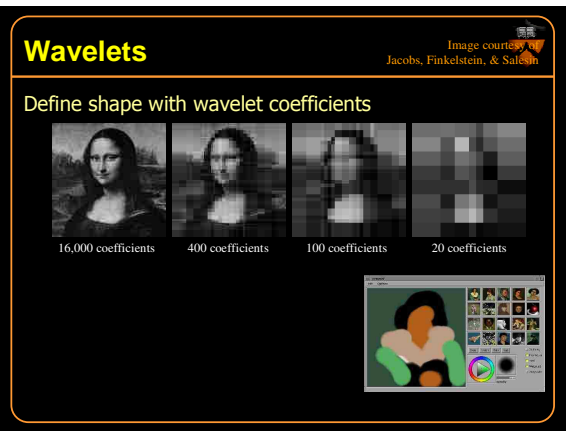
Ranked Matches







- ### Voxels
- Properties
- ü Discriminating
  - ü Insensitive to noise
  - ü Insensitive to topology
  - ü Robust to degeneracies
  - ü Quick to compute
  - Efficient to match?
  - X Concise to store
  - X Invariant to transforms



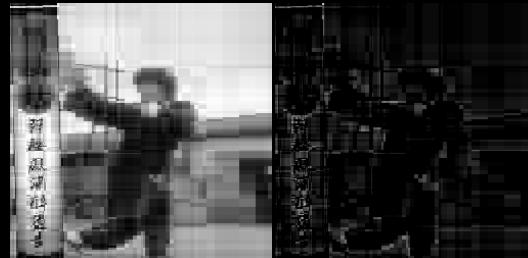
- ### Wavelets
- Jacobs, Finkelstein, & Salesin SIGGRAPH 95
- Descriptor 1:
- Given an  $N \times N \times N$  grid, generate an  $N \times N \times N$  array of the wavelet coefficients for the standard Haar basis functions
- Descriptor 2:
- Truncate: Find the  $m$  largest coefficients and set all others equal to zero
  - Quantize: Set the non-zero coefficients to +1 or -1 depending on their sign



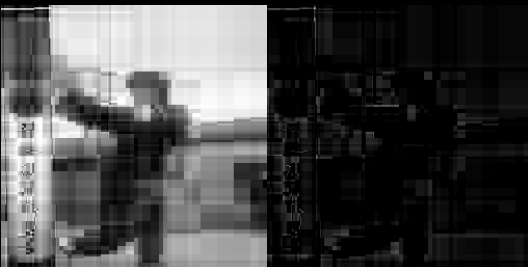
**Truncated And Quantized to 5000**



**Truncated And Quantized to 1000**



**Truncated And Quantized to 500**



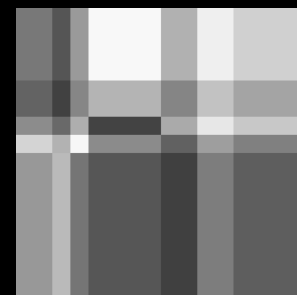
**Truncated 100**



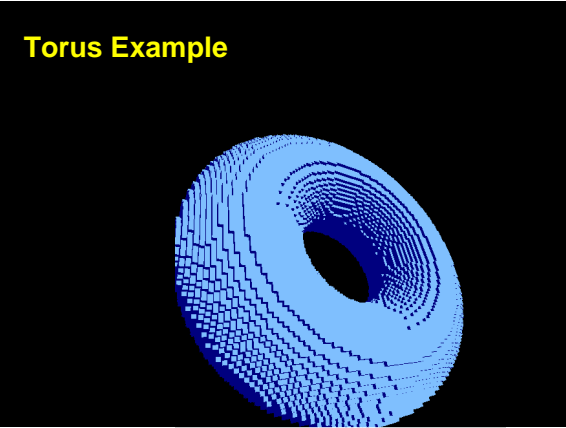
**Truncated 50**



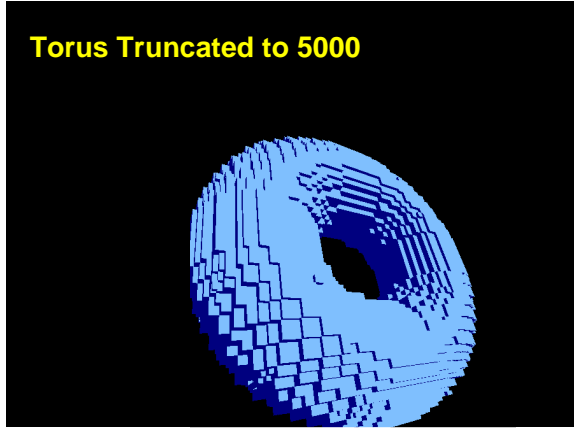
**Truncated 10**



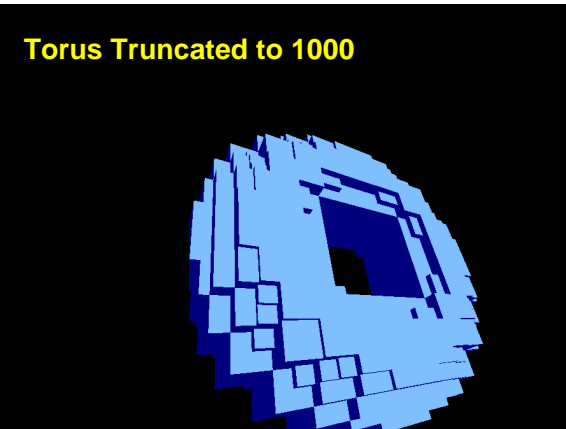
**Torus Example**



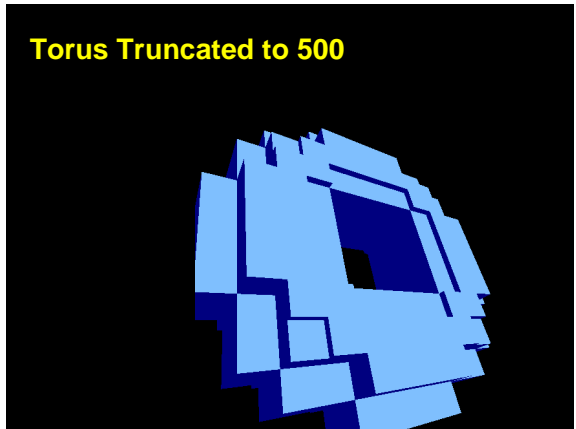
**Torus Truncated to 5000**



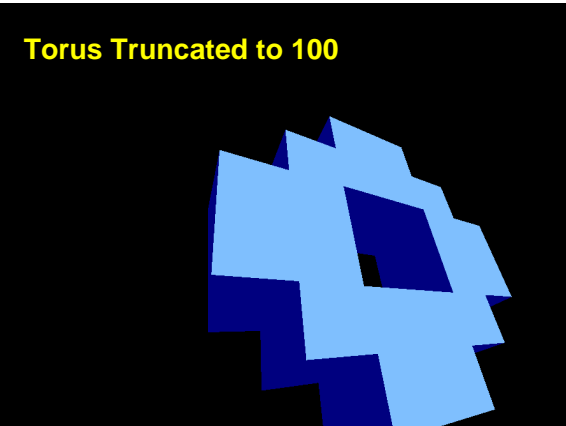
**Torus Truncated to 1000**



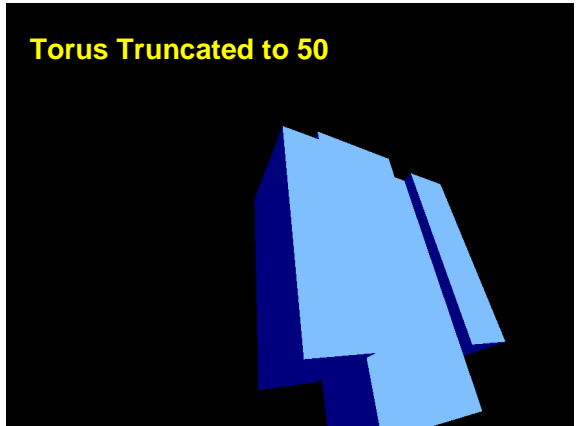
**Torus Truncated to 500**



**Torus Truncated to 100**



**Torus Truncated to 50**



## Wavelets

Jacobs, Finkelstein, & Sales  
SIGGRAPH 95

### Distance Function 1:

- The query metric is defined by:

$$d(A, B) = \sum_{i,j,k} w_{i,j,k} \|A[i, j, k] - B[i, j, k]\|$$

where  $A[i,j,k]$  and  $B[i,j,k]$  are the truncated and quantized coefficients and  $w_{i,j,k}$  are weights, fine tuned to the database.

## Wavelets

Jacobs, Finkelstein, & Sales  
SIGGRAPH 95

### Properties

- Insensitive to noise
- Insensitive to topology
- Robust to degeneracies
- Quick to compute
- Efficient to match
- Concise to store
- Discriminating?
- X Invariant to transforms**



## Moments

Define shape by moments of inertia:

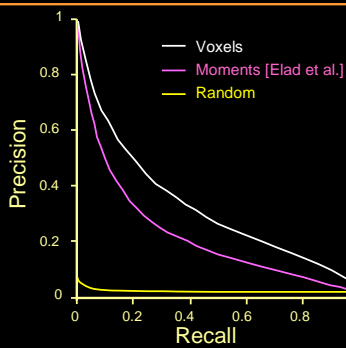
$$m_{pqr} = \int_{\text{surface}} x^p y^q z^r dx dy dz$$

## Moments Retrieval Experiment

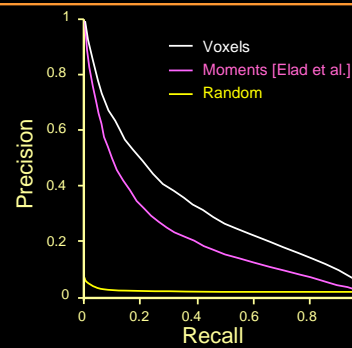
Test database is Viewpoint household collection  
1,890 models, 85 classes



## Moments Retrieval Results



## Moments Retrieval Results



## Moments

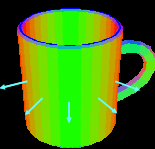
Properties

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- ü Concise to store
- X Insensitive to noise
- X Invariant to transforms
- X Discriminating


## Extended Gaussian Image

Define shape with histogram of normal directions

- Invertible for convex objects
- Spherical function





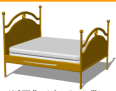





3D Model



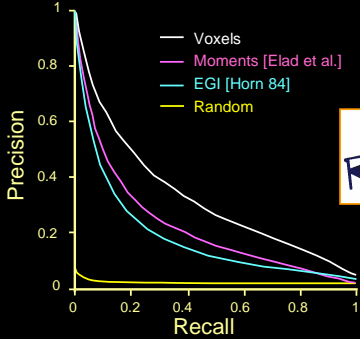
EGI

## EGI Retrieval Experiment

Test database is Viewpoint household collection  
1,890 models, 85 classes


 153 dining chairs	 25 livingroom chairs	 16 beds	 12 dining tables
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## EGI Retrieval Results



Legend:

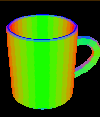

- Voxels
- Moments [Elad et al.]
- EGI [Horn 84]
- Random

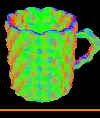



## Extended Gaussian Images


Properties

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- ü Quick to compute
- ü Efficient to match
- ü Concise to store
- X Insensitive to noise
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- X Invariant to transforms
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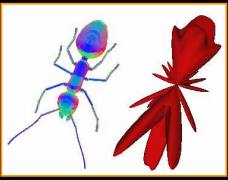



## Other Rotation-Dependent Descriptors



Spherical Extent Functions  
(Vranic & Saupe, 2000)



Shape Histograms (sectors)  
(Ankerst, 1999)



## Statistical Shape Descriptors

### Alignment-dependent

- Voxels
- Wavelets
- Moments
- Extended Gaussian Image
- Spherical Extent Function
- Spherical Attribute Image

### Alignment-independent

- Shape distributions
- Shape histograms
- Harmonic descriptor

## Alignment

### Translation (*Center of Mass*)

$$c = \frac{1}{n} \sum_{i=1}^n p_i$$

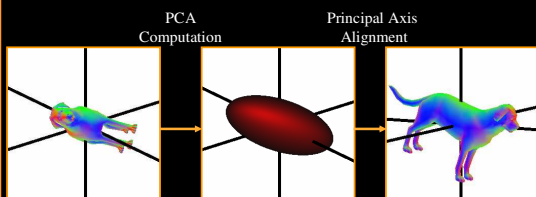
### Scale (*Radial Deviation*)

$$s = \sqrt{\frac{1}{n} \sum_{i=1}^n \|p_i\|^2}$$

## Alignment

### Rotation (PCA)

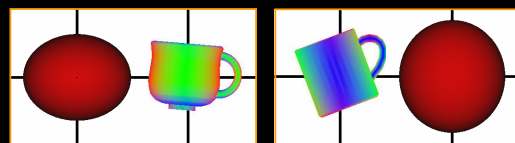
- *Principal axes are eigenvectors associated with largest eigenvalues of 2nd order moments covariance matrix*



## Alignment

### Rotation (PCA)

- *Principal axes are eigenvectors associated with largest eigenvalues of 2nd order moments covariance matrix*

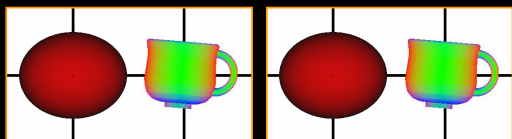


Not very robust!

## Alignment

### Mirror

- *PCA does not give directions for principal axes*



Need heuristics to determine positive axes!

## Alignment-Independent Descriptors

Observation: it is difficult to normalize for differences in rotation and mirroring



Three mugs aligned automatically with PCA

Motivation: build a shape descriptor that is invariant to rotations and mirrors and as discriminating as possible

## Shape Histograms

Image courtesy of Ankerst et al., 1999

Shape descriptor stores histogram of how much surface resides at different radii from center of mass

Shape Histograms (shells)  
(Ankerst, 1999)

## Shape Histograms

Image courtesy of Misha Kazhdan

Shape descriptor stores histogram of how much surface resides at different radii from center of mass

3D Model → Spherical Decomposition → Shape Descriptor

## Shape Histogram Experiment

Test database is Viewpoint household collection  
1,890 models, 85 classes

153 dining chairs    25 livingroom chairs    16 beds    12 dining tables  
8 chests    28 bottles    39 vases    36 end tables

## Shape Histogram Retrieval Results

Precision-recall curves (mean for all queries)

Legend:  
— Shape Histogram [Ankerst et al.]  
— EGI [Horn]  
— Moments [Elad et al.]  
— Random

## Shape Histograms

Properties

- ü Insensitive to noise
- ü Insensitive to topology
- ü Robust to degeneracies
- ü Quick to compute
- ü Efficient to match
- ü Concise to store
- ü Invariant to rotations
- Discriminating?

## Harmonic Shape Descriptor


Key idea:

- Decompose each sphere into irreducible set of rotation independent components
- Store "how much" of the model resides in each component

3D Model → Harmonic Decompositions → Shape Descriptor

### Step 1: Normalization

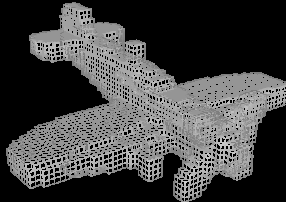
Normalize for translation and scale



3D Model

### Step 2: Voxelization

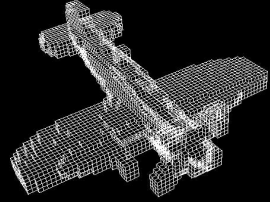
Rasterize polygon surfaces into 3D voxel grid



3D Voxel Grid

### Step 3: Spherical Decomposition


Intersect with concentric spheres



Spherical Functions

### Step 4: Frequency Decomposition


Represent each spherical function as a sum of harmonic frequencies (orders)



Spherical Functions

### Step 4: Frequency Decomposition

Represent each spherical function as a sum of harmonic frequencies (orders)

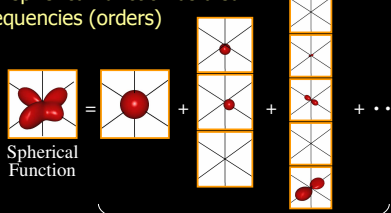


Spherical Function

Spherical Functions

### Step 4: Frequency Decomposition

Represent each spherical function as a sum of harmonic frequencies (orders)



Spherical Function

Harmonic Decomposition

### Step 4: Frequency Decomposition

Represent each spherical function as a sum of harmonic frequencies (orders)

Spherical Function = Constant + 1<sup>st</sup> Order + 2<sup>nd</sup> Order + ...

### Step 4: Frequency Decomposition

Represent each spherical function as a sum of harmonic frequencies (orders)

Amplitudes are invariant to rotation

Spherical Function = Frequency Decomposition

### Step 5: Amplitude Computation

Store "how much" ( $L_2$ -norm) of the shape resides in each harmonic frequency of each sphere

Harmonic Shape Descriptor

### Matching Harmonic Descriptors

Define similarity as  $L_2$ -distance between descriptors

- Enables nearest neighbor indexing and fast search
- Provides lower bound for  $L_2$ -distance between models

Sim [Descriptor 1, Descriptor 2] = ||Descriptor 1 - Descriptor 2||

### Harmonic Shape Descriptor

Properties

- Ø Concise to store?
  - Quick to compute?
  - Insensitive to noise?
  - Insensitive to topology?
  - Robust to degeneracies?
  - Invariant to transforms?
  - Efficient to match?
  - Discriminating?

2048 bytes per model  
(16 frequencies x 32 radii x 4 bytes)

### Harmonic Shape Descriptor

Properties

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- Ø Quick to compute?
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1.6 seconds (on average)

Polygons  
Voxels  
Spherical Decomposition  
Frequency Decomposition  
Harmonic Shape Descriptor

## Harmonic Shape Descriptor

Properties

- ü Concise to store
- Ø Quick to compute?
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Polygons  
Voxels  
Spherical Decomposition  
Frequency Decomposition  
Harmonic Shape Descriptor

## Harmonic Shape Descriptor

Properties

- ü Concise to store
- ü Quick to compute
- Ø Insensitive to noise
- Ø Insensitive to topology
- Ø Robust to degeneracies
  - Invariant to transforms?
  - Efficient to match?
  - Discriminating?

Rasterize polygon surfaces  
(no solid reconstruction)

## Harmonic Shape Descriptor

Properties

- ü Concise to store
- ü Quick to compute
- ü Insensitive to noise
- ü Insensitive to topology
- ü Robust to degeneracies
  - ü Rotation
  - ü Mirror
  - ü Translation (w/ normalization)
  - ü Scale (w/ normalization)
- Ø Invariant to transforms
  - Efficient to match?
  - Discriminating?

## Harmonic Shape Descriptor

Properties

- ü Concise to store
- ü Quick to compute
- ü Insensitive to noise
- ü Insensitive to topology
- ü Robust to degeneracies
- ü Invariant to transforms
- Ø Efficient to match?
  - Discriminating?

0.23 seconds to search 17,500 models

Search time (secs)

Database size (models)

## Harmonic Shape Descriptor

Properties

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- ü Quick to compute
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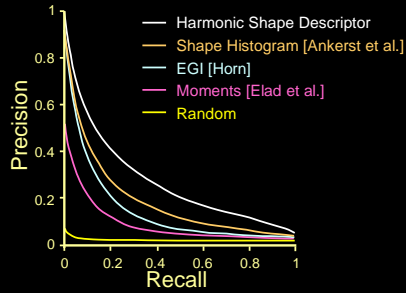
## Harmonic Matching Results

Test database is Viewpoint household collection  
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8 chests	28 bottles	39 vases	36 end tables

## Harmonic Retrieval Results

Precision-recall curves (mean for all queries)



## 3D Representations for Retrieval

Statistical shape descriptors

- Voxels, moments, wavelets, ...
- Attributes, histograms, ...

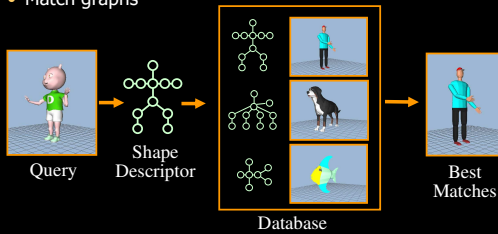
Structural shape descriptors

- Feature-based methods
- Part-based methods
- Skeletons

## Structural Shape Descriptors

General Approach:

- Construct graph where nodes represent parts and edges represent relationships between parts
- Match graphs



## Structural Shape Descriptors

Graph construction

- Local features
- Primitives
- Skeletons

Graph matching

- Combinatorial methods
- Optimization methods
- Algebraic methods

## Structural Shape Descriptors

Graph construction

∅ Local features

- Primitives
- Skeletons

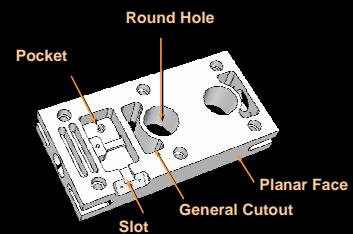
Graph matching

- Combinatorial methods
- Optimization methods
- Algebraic methods

## Local Features

Image courtesy of Bill Roloff

Construct graph representing geometric relationship between features in 3D shape

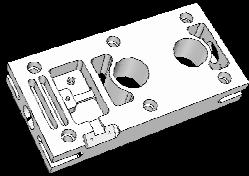


## Local Features

Images courtesy of Bill Ragan

General strategy

- Extract features
- Construct graph
- Match graphs

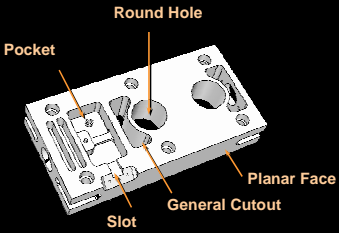


## Local Features

Images courtesy of Bill Ragan

Example 1

- ∅ Extract features
- Construct graph
- Match graphs

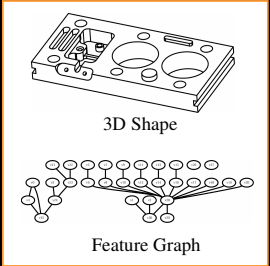


## Local Features

Images courtesy of Bill Ragan

Example 1

- Extract features
- ∅ Construct graph
- Match graphs



3D Shape

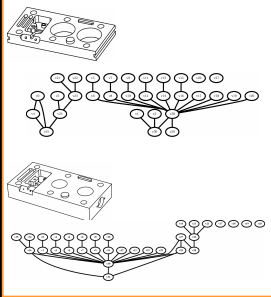
Feature Graph

## Local Features

Images courtesy of Bill Ragan

Example 1


- Extract features
- Construct graph
- ∅ Match graphs



## Local Features

Example 2

- Extract features
- Construct graph
- Match graphs



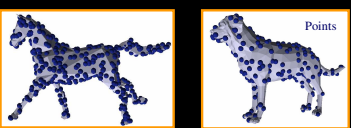
A

B Surface

## Local Features

Example 2

- ∅ Extract features
- Construct graph
- Match graphs



A

B Points

## Local Features

Example 2

- ∅ Extract features
- Construct graph
- Match graphs

## Local Features

Example 2

- Extract features
- ∅ Construct graph
- Match graphs

## Local Features

Example 2

- Extract features
- Construct graph
- ∅ Match graphs

$$D(A,B) = \sum_{\text{Correspondences}} \Delta\text{FeatureShape} + \sum_{\text{Correspondence Pairs}} \Delta\text{SpatialConsistency}$$

## Local Features

Example 2

- Extract features
- Construct graph
- ∅ Match graphs

$$D(A,B) = \sum_{\text{Correspondences}} \Delta\text{FeatureShape} + \sum_{\text{Correspondence Pairs}} \Delta\text{SpatialConsistency}$$

## Local Features

Properties

- Decomposes shape into graph based on features
- Features can be prioritized, pruned, and attributed
- Invariance to transformations

Limitations

- Robust to missing or extra parts
- Computationally expensive

## Structural Shape Descriptors

Graph construction

- Local features
- ∅ Primitives
- Skeletons

Graph matching

- Combinatorial methods
- Optimization methods
- Algebraic methods



### Primitive Graphs

Image courtesy of Robert Orosz

Decompose shape into parts by "covering" it with simple primitives

### Primitive Graphs

Image courtesy of Patrick Min

Decompose shape into parts by "covering" it with simple primitives

- Provides convenient parameterization for reasoning about variability within a class for some application domains

### Primitive Graphs

Image courtesy of Patrick Min

General strategy

- Fit primitives to main parts of shape
- Build graph representing primitives
- Match graphs

### Primitive Graphs

Design choices

- Which primitives?
- How fit?

### Primitive Graphs

Image courtesy of Patrick Min

Design choices

- Which primitives?
- How fit?

more expressive

Ellipses, cylinders, ...  
Superquadrics  
Geons  
Generalized Cylinders

more parameters

### Primitive Graphs

Image courtesy of Robert Orosz

Design choices

- Which primitives?
- How fit?

While not done  
Insert best fitting primitive  
Adjust previous primitives

## Primitive Graphs

Images courtesy of Robert Ossola

Example: Teddy

The image shows a brown teddy bear on the left. To its right is a 3D wireframe graph of the bear, composed of various cylinders and spheres. Further right are several individual primitive shapes (cylinders and spheres) that were used to construct the bear's graph.

## Primitive Graphs

Images courtesy of Robert Ossola

Example: Dog

The image shows a grey dog on the left. To its right is a 3D wireframe graph of the dog, composed of various cylinders and spheres. Further right are several individual primitive shapes (cylinders and spheres) that were used to construct the dog's graph.

## Primitive Graphs

Images courtesy of Robert Ossola

Properties

- Decomposes shape into graph based on primitive parts
- Primitives can be prioritized, pruned, and attributed
- Invariance to transformations

Limitations

- Computationally expensive
- Sensitive to noise
- Sensitive to primitive order & stopping criteria

The image shows two white horses. The horse on the right has a primitive graph overlay, with yellow circles highlighting specific parts of the graph, illustrating the sensitivity to primitive order and stopping criteria.

## Structural Shape Descriptors

Graph construction

- Local features
- Primitives
- Skeletons

Graph matching

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- Optimization methods
- Algebraic methods

## Skeletons

Image courtesy of Nystrom

Medial Axis [Blum 1967]

The image shows a white horse on the left. To its right is a diagram of a leaf with its medial axis highlighted in black. The diagram is labeled with numbers 1, 2, and 3, corresponding to the legend:

- ① first order skeleton
- ② second order skeleton
- ③ third order skeleton

"A Transform for Extracting New Descriptors of Shape"

## Skeletons

Images courtesy of Patrick Min

Medial Axis [Blum 1967]

- Locus of centers of maximal balls
- Locus of points equidistant from surface
- Local maxima in distance transform

The diagram shows a grey irregular shape on the left. Inside it, a white line represents the skeleton. Several white circles of varying sizes are shown, with arrows pointing to their centers, labeled as 'maximal circle'. To the right, a 3D grid of points is shown, with a blue line representing the skeleton, labeled as 'boundary' and 'skeleton'.

## Skeletons

Images courtesy of Siddiqui

Shock Graph

- Characterize regions of the skeleton as first to fourth order shocks (protrusions, necks, bends and seeds)

## Skeletons

Images courtesy of Siddiqui

Shock Graph Example

## Skeletons

Image courtesy of Kimura

More complex in 3D than 2D

## Skeletons

Image courtesy of Amalia

More complex in 3D than 2D

- Medial surface has 1D curves and 2D sheets in 3D

## Skeletons

Image courtesy of Li & Silver

More complex in 3D than 2D

- Medial surface has 1D curves and 2D sheets in 3D
- Medial surface is often approximated by centerlines

## Skeletons

Images courtesy of Silver & Li et al.

Computational methods

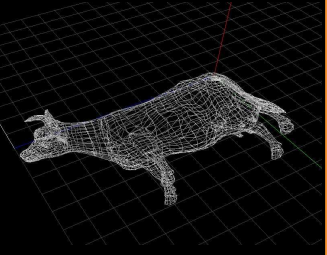
- Thinning
- Voronoi
- Distance transform
- Grassfire
- Simplification

## Skeletons

Images courtesy of Patrick Min

Example: thinning

- Start with mesh
- Voxelize
- Thin voxels
- Make graph
- Simplify graph
- Assign attributes
- Match graphs

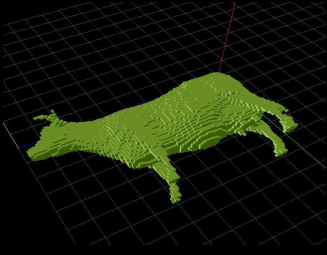


## Skeletons

Images courtesy of Patrick Min

Example: topological thinning

- Start with mesh
- Voxelize
- Thin voxels
- Make graph
- Simplify graph
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- Match graphs

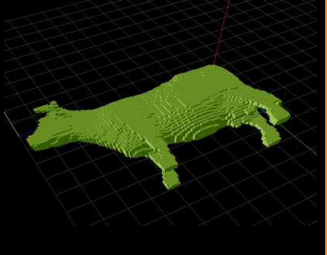


## Skeletons

Images courtesy of Patrick Min

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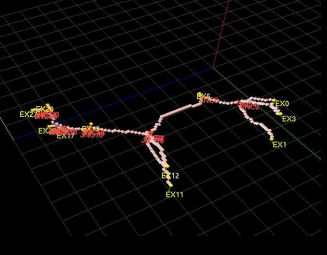


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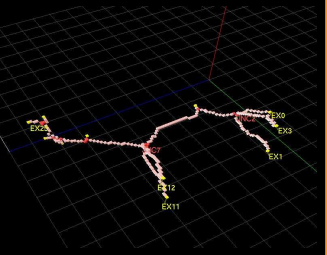


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Images courtesy of Patrick Min

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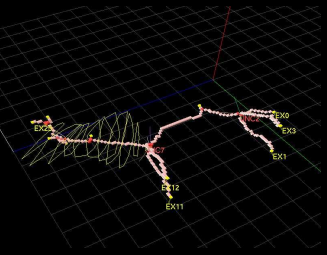


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Images courtesy of Patrick Min

Example: topological thinning

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## Skeletons

Images courtesy Patrick Min

Example: topological thinning

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- Make graph
- Simplify graph
- Assign attributes
- Match graphs

## Skeletons

Images courtesy Deborah Silver

Branches can be prioritized and/or pruned

## Skeletons

Images courtesy Deborah Silver

## Skeletons

Images courtesy Deborah Silver

## Skeletons

Properties

- Decomposes shape into graph based on local symmetries
- Branches can be prioritized, pruned, and attributed
- Invariance to transformations

Limitations

- Computationally expensive
- Sensitive to noise
- May yield complex structures

## Graph Construction Summary

Construct graph where nodes represent parts and edges represent relationships between parts

- Features
- Primitives
- Skeletons

Issues

- Computationally expensive
- Sensitive to noise
- Sensitive to stopping criteria
- Often do not produce same topology for similar shapes

## Structural Shape Descriptors

Graph construction

- Skeletons
- Primitive graphs
- Feature graphs
- Surface segmentation graphs

Graph matching

- Combinatorial methods
- Optimization methods
- Algebraic methods

## Structural Shape Descriptors

Graph construction

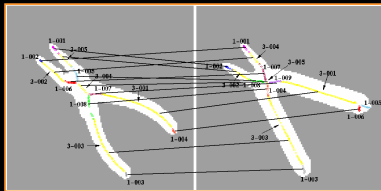
- Skeletons
- Primitive graphs
- Feature graphs
- Surface segmentation graphs

Graph matching

- ∅ Combinatorial methods
- Optimization methods
- Algebraic methods

## Combinatorial Graph Matching

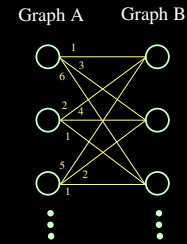
Find a matching (node correspondence) between two graphs with greatest total weight



## Combinatorial Graph Matching

Find a matching (node correspondence) between two graphs with greatest total weight

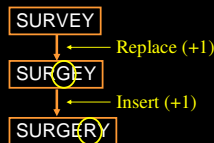
- ∅ Bipartite matching
- Graph edit distance



## Combinatorial Graph Matching

Find a matching (node correspondence) between two graphs with greatest total weight

- Bipartite matching
- ∅ Graph edit distance



Total Edit Cost (+2)

## Graph Matching Summary

Combinatorial methods

- No indexing
- Optimization methods

- No indexing

Algebraic methods (spectral methods)

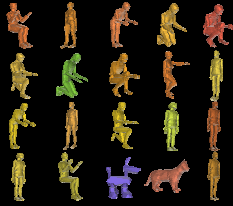
- Limited many-to-many matching support

## Structural Shape Descriptors

Images courtesy of 

### Questions:

- Is structural matching right for application?
- Topological or geometric matching?
- Extent and type of intra-class variation?
- Extent and type of noise?
- Surface degeneracies?
- Computational speed?
- Indexing required?



## 3D Representations for Retrieval



### Statistical shape descriptors

- Voxels, moments, wavelets, ...
- Attributes, histograms, ...

### Structural representations

- Local features
- Primitives
- Skeletons