COS 521: Advanced Algorithm Design
Homework 3
Due: Wed, Nov 22 (in class)

Collaboration Policy: You may collaborate with other students on these problems. Collaboration is limited to discussion of ideas only, and you should write up the solutions entirely *on your own* and *list your collaborators* as well as *cite any references* you may have used.

1. A cycle cover of a graph is a collection of cycles such that every vertex is contained in exactly one cycle. This applies to both directed and undirected graphs. In a cycle cover of a directed graph, every vertex has exactly one incoming and one outgoing edge. In a cycle cover of an undirected graph, every vertex has exactly two edges incident on it. Give a polynomial time algorithm to find a cycle cover in a graph (either directed or undirected) or prove that none exists. *Hint:* use matching.

2. Prove von Neumann's minimax theorem without using LP duality. *Hint*: use the Multiplicative Weights Update algorithm to design good strategies for the two players.

3. Show that the perceptron algorithm can be used to solve arbitrary linear programs. *Hint:* show a reduction from an arbitrary linear program to the linear separator problem.

4. Consider the following optimization problem with *robust conditions*:

$$\min\{c^T x : x \in \Re^n; Ax \geq b \text{ for any } A \in F\},$$

where $b \in \Re^m$ and F is a set of $m \times n$ matrices:

$$F = \{A; \forall i, j; a_{ij}^{min} \leq a_{ij} \leq a_{ij}^{max}\}.$$

   (a) Considering F as a polytope in $\Re^{m \times n}$, what are the vertices of F ?

   (b) Show that instead of conditions for all $A \in F$, it is enough to consider the vertices of F. Write the resulting linear program. What is its size ? Is this poynomial in the size of the input namely m, n and the sizes of b, c, a_{ij}^{min} and a_{ij}^{max} ?

   (c) Derive a more efficient description of the linear program: Write the conditions on x given by one row of A, for all choices of A. Formulate the condition as a linear program. Use duality and formulate the original problem as a linear program. What is the size of this one ? Is this polynomial in the size of the input ?