1. Show that PSPACE is closed under the operations union, complementation, and star.

2. The Japanese game go-moku is played by two players, “X” and “O” on a 19 × 19 grid. Players take turns placing markers, and the first player to achieve 5 of his markers consecutively in a row, column, or diagonal, is the winner. Consider this game generalized to an n × n board. Let

\[ GM = \{ \langle B \rangle \mid B \text{ is a position in generalized go-moku, where player “X” has a winning strategy} \} \]

By a position we mean a board with markers placed on it, such as may occur in the middle of a play of the game, together with an indication of which player moves next. Show that \( GM \in \text{PSPACE}. \)

3. Show that, if every NP-hard language is also PSPACE-hard, then PSPACE = NP.

4. Let A be the language of properly nested parentheses. For example, (()) and (((())())) are in A, but ) is not. Show that A is in L.

5. (a) Let ADD = \{ \langle x, y, z \rangle \mid x, y, z > 0 \text{ are binary integers and } x + y = z \}. Show that ADD ∈ L.

(b) Let PAL-ADD = \{ \langle x, y \rangle \mid x, y > 0 \text{ are binary integers where } x + y \text{ is an integer whose binary representation is a palindrome} \}. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse). Show that PAL-ADD ∈ L.

6. (Optional) Show that 2SAT is NL-complete.