1. Show that \( EQ_{CFG} \) is co-Turing-recognizable. (Look at the definition of \( EQ_{CFG} \) in the book.)

2. If \( A \preceq_m B \) and \( B \) is a regular language, does that imply that \( A \) is a regular language? Why or why not?

3. Let \( \Gamma = \{a, 1, \sqcup\} \) be the tape alphabet for all TMs in this problem. Define the busy beaver function \( BB : \mathbb{N} \to \mathbb{N} \) as follows. For each value of \( k \), consider all \( k \)-state TMs that halt when started with a blank tape. Let \( BB(k) \) be the maximum number of 1s that remain on the tape among all of these machines. Show that \( BB \) is not a computable function.

4. Let

\[
 f(x) = \begin{cases} 
 3x + 1 & \text{for odd } x \\
 x/2 & \text{for even } x 
\end{cases}
\]

for any natural number \( x \). If you start with an integer \( x \) and iterate \( f \), you obtain a sequence, \( x, f(x), f(f(x)), \ldots \). Stop if you ever hit 1. For example, if \( x = 17 \), you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But, the question of whether all positive starting points end up at 1 is unsolved; it is called the \( 3x + 1 \) problem. Suppose that \( A_{TM} \) were decidable by a TM \( H \). Use \( H \) to describe a TM that is guaranteed to state the answer to the \( 3x + 1 \) problem.

5. Let \( S = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \{\langle M\rangle\}\} \). Show that neither \( S \) nor \( \bar{S} \) is Turing-recognizable.

6. (Optional) Prove that there exist two languages \( A \) and \( B \) that are Turing-incomparable, i.e. where \( A \nless_T B \) and \( B \nless_T A \).