1. Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

2. Let \(\text{INFINITE}_{PDA} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}\). Show that \(\text{INFINITE}_{PDA}\) is decidable.

3. Let \(A = \{\langle R \rangle \mid R \text{ is a regular expression describing a language containing at least one string } w \text{ that has } 111 \text{ as a substring (i.e., } w = x111y \text{ for some } x \text{ and } y)\}\). Show that \(A\) is decidable.

4. Let \(S = \{\langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}\). Show that \(S\) is decidable.

5. A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

6. (Optional) Let \(E = \{\langle M \rangle \mid M \text{ is a DFA that accepts some string with more } 1\text{s than } 0\text{s}\}\). Show that \(E\) is decidable. (Hint: Theorems about CFLs are helpful here.)