COS487 Theory of Computation **Fall 2006**

Assignment #2

Due: Thursday October 5 *Sean Hallgren*

1. An ***all*-NFA** M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if *every* possible state that M could be in after reading input x is a state from F . Note, in contrast, that an ordinary **NFA** accepts a string if *some* state among these possible states is an accept state. Prove that ***all*-NFA**s recognize the class of regular languages.

2. Show that the language $L = \{0^p \mid p \text{ is a prime}\}$ is not regular.

3. Let $\Sigma = \{0, 1, +, =\}$ and

   $$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

   Show that ADD is not regular.

4. Give context-free grammars that generate the following languages. In all parts the alphabet is $\Sigma = \{0, 1\}$.

   (a) $\{w \mid$ the length of w is odd$\}$

   (b) $\{w \mid w = w^R$, that is w is palindrome$\}$

   (c) The empty set

5. Give informal descriptions and state diagrams of pushdown automata for the languages in **Problem 4**. The formal 6-tuple is not required.

6. (Optional) If A is any language, let $A_{\frac{1}{3}-\frac{1}{3}}$ be the set of all strings in A with their middle thirds removed so that

   $$A_{\frac{1}{3}-\frac{1}{3}} = \{xz \mid \text{for some } y, |x| = |y| = |z| \text{ and } xyz \in A\}$$

   Show that, if A is regular, then $A_{\frac{1}{3}-\frac{1}{3}}$ is not necessarily regular.