1. Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts $\Sigma = \{a, b\}$.

(a) $\{w \mid w \text{ contains neither the substrings } ab \text{ nor } ba\}$

(b) $\{w \mid w \text{ is any string not in } a^*b^*\}$

2. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is $\{0, 1\}$.

(a) The language $\{0\}$ with two states

(b) The language $0^*1^*0^+$ with three states

(c) The language $\{\epsilon\}$ with one state

3. Use the construction given in Theorem 1.39 (in textbook) to convert the following non-deterministic finite automaton to an equivalent deterministic finite automaton.

4. For any string $w = w_1w_2\cdots w_n$, the reverse of $w$, written $w^R$, is the string $w$ in reverse order, $w_n\cdots w_2w_1$. For any language $A$, let $A^R = \{w^R \mid w \in A\}$. Show that if $A$ is regular, so is $A^R$.

5. Let $\Sigma = \{0, 1\}$ and let

$$D = \{w \mid w \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}.$$

Thus $101 \in D$ because 101 contains a single 01 and a single 10, but $1010 \notin D$ because 1010 contains two 10s and one 01. Show that $D$ is a regular language.

6. (Optional) If $A$ is any language, let $A_{\frac{1}{2}}$ be the set of all first halves of strings in $A$ so that

$$A_{\frac{1}{2}} = \{x \mid \text{ for some } y, \ |x| = |y| \text{ and } xy \in A\}.$$  

Show that, if $A$ is regular, then so is $A_{\frac{1}{2}}$. 