COS487 Theory of Computation **Fall 2006**

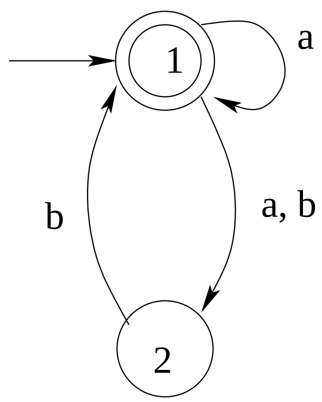
Assignment #1

Due: Thursday September 28 *Sean Hallgren*

1. Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts $\Sigma = \{a, b\}$.
   (a) $\{w \mid w$ contains neither the substrings ab nor ba$\}$
   (b) $\{w \mid w$ is any string not in $a^*b^*\}$

2. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is {0, 1}.
   (a) The language {0} with two states
   (b) The language $0^*1^*0^+$ with three states
   (c) The language $\{\epsilon\}$ with one state

3. Use the construction given in **Theorem 1.39** (in textbook) to convert the following nondeterministic finite automaton to an equivalent deterministic finite automaton.



4. For any string $w = w_1 w_2 \cdots w_n$, the *r*everse of w, written w^R, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

5. Let $\Sigma = \{0, 1\}$ and let
$$D = \{w \mid w \text{ contains an equal number of occurrences of the substrings 01 and 10}\}.$$
Thus $101 \in D$ because 101 contains a single 01 and a single 10, but $1010 \notin D$ because 1010 contains two 10s and one 01. Show that D is a regular language.

6. (Optional) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that
$$A_{\frac{1}{2}-} = \{x \mid \text{ for some } y,\ |x| = |y| \text{ and } xy \in A\}.$$
Show that, if A is regular, then so is $A_{\frac{1}{2}-}$.