Selection $\sigma_P(R)$ for relation $R$ and predicate $P$ on attributes of $R$:
is the relation with the same schema as $R$ that contains those tuples of $R$ that satisfy $P$.
Candidate keys and foreign keys in $R$ are preserved.

Projection $\pi_S(R)$ for relation $R$ and $S$ a list of attributes from $R$:
is the relation containing all tuples formed by taking a tuple from $R$ and keeping only the
attributes listed in $S$.
In the formal definition, relations are *sets*, and so duplicates are removed.
In practice, duplicates are not removed unless explicitly requested.
If a candidate key or foreign key is projected (i.e. included in $S$) then the constraint is preserved.
If no candidate key is projected, the only key may be all attributes in $S$ (in the set model).

Union $R \cup S$ for relations $R$ and $S$ on the same universe $D_1 \times D_2 \times \ldots \times D_k$,
where $D_i$ is the
domain for the $i^{th}$ attribute:
is the relation that includes any tuple in either $R$ or $S$.
Formal model removes duplicates.
Candidate keys are not preserved.
A foreign key is preserved if it is a foreign key for both $R$ and $S$ using corresponding attributes
and referencing the same relation.

Set difference $R - S$ for relations $R$ and $S$ on the same universe:
is the relation that includes all tuples in $R$ that are not in $S$.
*constraints left as an exercise*

Cross product $R \times T$ for relations $R$ and $T$:
For $R \subseteq D_1 \times D_2 \times \ldots \times D_k$ and $T \subseteq S_1 \times S_2 \times \ldots \times S_m$,
$R \times T \subseteq D_1 \times D_2 \times \ldots \times D_k \times S_1 \times S_2 \times \ldots \times S_m$ and tuple
$(d_1, d_2, \ldots, d_k, s_1, s_2, \ldots, s_m) \in R \times T$ if and only if
$(d_1, d_2, \ldots, d_k) \in R$ and $(s_1, s_2, \ldots, s_m) \in T$.
If attributes in positions $i_1, i_2, \ldots, i_\alpha$ form a candidate key for $R$ and attributes in positions $j_1, j_2, \ldots, j_\beta$ form a candidate key for $T$, then the union of the attributes - positions $i_1, i_2, \ldots, i_\alpha, k+j_1, k+j_2, \ldots, k+j_\beta$ of $R \times T$ - form a candidate key for $R \times T$.
Foreign keys for each of $R$ and $T$ are preserved using corresponding attributes of $R \times T$.

Renaming operation $\rho(Q(L), E)$, where $E$ is a relational algebra expression, $Q$ is a new
relation name and $L$ is a list of (old name $\rightarrow$ new name) or (attribute position $\rightarrow$ new name)
mappings of attributes of $E$:
defines relation $Q$ as the relation expressed by $E$, but with attributes in the list $L$ renamed
according to the given mappings.
All constraints on the relation expressed by $E$ are preserved with appropriate renaming of
attributes.