COS 341: Discrete Mathematics

Precept #6 Fall 2006
For the week of: November 13

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1. In class, we discussed the following game played between Alice and Bob. Alice secretly chooses two distinct numbers between 0 and n, and places them into two envelopes. One of the envelopes is chosen at random, and the contents of the selected envelope is revealed to Bob. Bob then needs to guess whether the number that remains hidden in the unopened envelope is larger or smaller than the one that was revealed. Bob wins if his guess is correct; otherwise, Alice wins.

In class, we showed that Bob can play this game in a way that allows him to win with probability at least $1/2 + 1/(2n)$. Is this the best possible? Either find a better strategy for Bob, or show that Alice has a strategy for choosing numbers that prevents Bob from doing better, regardless of what strategy he is using.

2. Suppose we are given a biased coin that when flipped produces *heads* with unknown probability p, where $0 < p < 1$. Show how a fair "coin flip" can be simulated; in other words, describe a random experiment that produces the events "*heads*" and "*tails*" with probability exactly $1/2$ each by looking at multiple flips.