COS 341: Discrete Mathematics

Precept #3 Fall 2006
For the week of: October 9

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1. In yet another variant of the Towers of Hanoi problem, disks can only be moved on each move in a "right rotate fashion." That is, in a single move, a disk on post #1 can only be moved to post #2, a disk on post #2 can only be moved to post #3, and a disk on post #3 can only be moved to post #1. The goal, as usual, is to move all of the disks from post #1 to post #3, without placing a larger disk on a smaller one. Find the number of moves needed to solve this version of the problem.

2. Let C_n denote the number of paths in the plane from $(0, 0)$ to (n, n) that only go up or to the right by single unit-length steps, and never go above the diagonal line $y = x$ (although they are allowed to touch that diagonal line).

a. Show that C_n satisfies the recurrence relation:

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-k-1}.$$

b. Show a bijection between these paths and ways of parenthesizing the product of $n+1$ numbers, $x_0 \cdot x_1 \cdot x_2 \cdot \cdots \cdot x_n$. For instance, when $n = 3$, $x_0 \cdot x_1 \cdot x_2 \cdot x_3$ can be parenthesized as follows:

$$((x_0 \cdot x_1) \cdot x_2) \cdot x_3$$
$$(x_0 \cdot (x_1 \cdot x_2)) \cdot x_3$$
$$(x_0 \cdot x_1) \cdot (x_2 \cdot x_3)$$
$$x_0 \cdot ((x_1 \cdot x_2) \cdot x_3)$$
$$x_0 \cdot (x_1 \cdot (x_2 \cdot x_3))$$