COS 341: Discrete Mathematics

Precept #2 — Fall 2006

For the week of: October 2

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1. At a meeting, a group of n people, numbered $1, 2, \ldots, n$, are sitting around a circular table. Counting off, beginning with person 1, they one-by-one get up and leave the meeting, skipping every second person among those still remaining, until there is only one person left. For instance, if $n = 4$, then first person 2 leaves (skipping person 1), then person 4 (skipping person 3), then person 3 (skipping person 1), leaving only person 1. Let $J(n)$ denote the number of the last person remaining. Find $J(n)$, and prove the correctness of your answer.

2. In the following, you can assume throughout that "nothing bad happens," i.e., that no two lines or planes are parallel, no three lines intersect at a single point, etc.

a. Find the number of regions that the plane is divided into by n lines.

b. Find the number of regions that space is divided into by n planes.

c. Sketch a generalization to more than three dimensions.

3. Consider the following recursive algorithm for multiplying two $2n$-bit numbers, a and b. Let A_0 be the number represented by the n lower-order bits of a, and let A_1 be the number represented by the n upper-order bits of a. Similarly define B_0 and B_1. Then

$$a = 2^n A_1 + A_0$$

and

$$b = 2^n B_1 + B_0.$$

a. Write the product ab as a linear combination of: $A_1 B_1$, $(A_1 - A_0)(B_0 - B_1)$ and $A_0 B_0$.

b. Use your answer above to derive a recursive algorithm for multiplying two numbers.

c. Let $T(n)$ denote the number of bit operations when multiplying two n-bit numbers. Derive a recurrence for $T(n)$.

d. Solve for $T(n)$. How does this compare to the conventional algorithm for multiplication?