3.2 Creating Data Types

Data Types

Data type. Set of values and operations on those values.

Built-in "primitive" types.

Data Type	Set of Values	Some Operations
`boolean`	`true`, `false`	not, and, or, xor
`int`	-2^{31} to $2^{31} - 1$	add, subtract, multiply
`String`	any sequence of characters	concatenate, compare

Last time. Write programs that use data types.
Today. Write programs to create our own data types.

Defining Data Types in Java

To define a data type, define:

- Set of values.
- Operations defined on them.

Java class. Allows us to define data types by specifying:

- Instance variables. [set of values]
- Methods. [operations defined on them]
- Constructors. [create and initialize new objects]

Point Charge Data Type

Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

Operations.

- Create a new point charge at (x_0, y_0) with electric charge q.
- Determine electric potential at (x, y) due to point charge.
- Convert to string.

$$V = k \frac{q}{r}$$

r = distance between (x, y) and (x_0, y_0)
k = electrostatic constant = $8.99 \times 10^9 \ N \cdot m^2 / C^2$

Charge Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

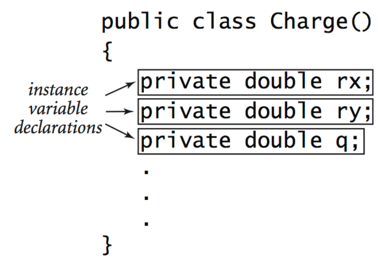
```
public static void main(String[] args) {
   double x = Double.parseDouble(args[0]);
   double y = Double.parseDouble(args[1]);
   Charge c1 = new Charge(.51, .63, 21.3);
   Charge c2 = new Charge(.13, .94, 81.9);
   double v1 = c1.potentialAt(x, y);
   double v2 = c2.potentialAt(x, y);
   System.out.println(c1);
   System.out.println(c2);
   System.out.println(v1 + v2);
}
```

```
% java Charge .50 .50
21.3 at (0.51, 0.63)
81.0 at (0.13, 0.94)
2.74936907085912E12
```


Anatomy of Instance Variables

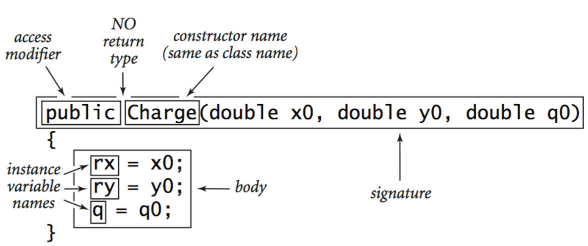
Instance variables. Specifies the set of values.

- Declare outside any method.
- Always use access modifier `private`. [stay tuned]



Anatomy of a Constructor

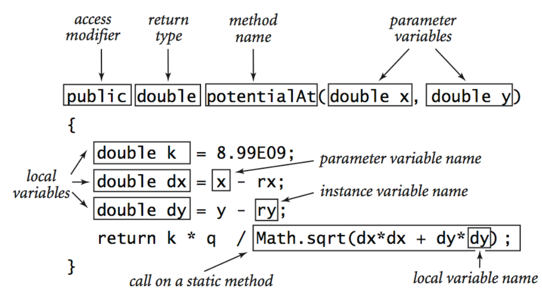
Constructor. Invoke with `new` to create new objects.




```
Charge c1 = new Charge(.51, .63, 21.3);
Charge c2 = new Charge(.13, .94, 81.9);
```

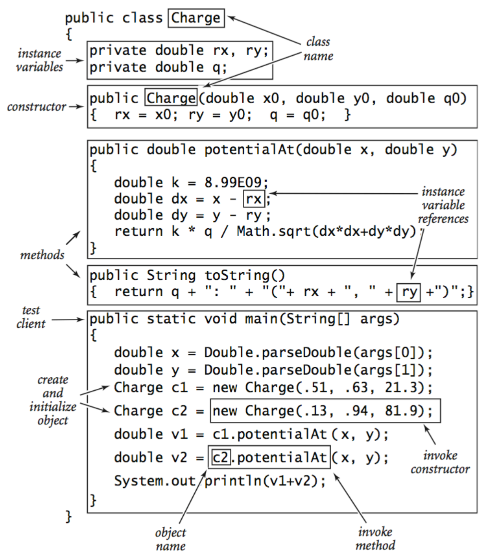

Anatomy of a Data Type Method

Method. Define operations on instance variables.




```
double v1 = c1.potentialAt(x, y);
double v2 = c2.potentialAt(x, y);
```

Anatomy of a Class

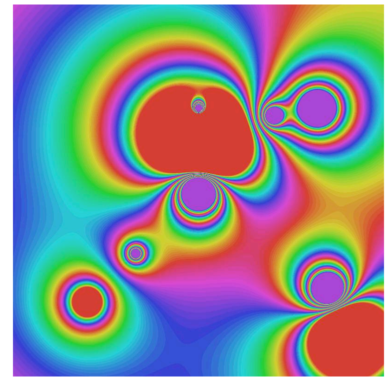


Potential Visualization

Potential visualization. Read in N point charges from a file; compute total potential at each point in unit square.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
% java Potential < charges.txt
```



Potential Visualization

Read in data.

```
int N = StdIn.readInt();
Charge[] a = new Charge[N];
for (int k = 0; k < N; k++) {
   double x0 = StdIn.readDouble();
   double y0 = StdIn.readDouble();
   double q0 = StdIn.readDouble();
   a[k] = new Charge(x0, y0, q0);
}
```

Arrays of objects. Allocate memory for the array; then allocate memory for each individual object.

Potential Visualization

Plot the data.

```
int SIZE = 512;
Picture pic = new Picture(SIZE, SIZE);
for (int i = 0; i < SIZE; i++) {
   for (int j = 0; j < SIZE; j++) {
      double V = 0.0;
      for (int k = 0; k < N; k++) {
         double x = 1.0 * i / SIZE;
         double y = 1.0 * j / SIZE;
         V += a[k].potentialAt(x, y);
      }
      Color color = getColor(V);
      pic.set(i, SIZE-1-j, color);
   }
}
pic.show();
```

$$V = \sum_i (k\, q_i / r_i)$$

compute color as a function of V

(0, 0) is upper left

Complex Numbers

Complex Number Data Type

Goal. Create a data type to manipulate complex numbers.
Set of values. Two real numbers. [real and imaginary parts]

API. `public class Complex` (PROGRAM 3.2.2)

	`Complex(double real, double imag)`	
`Complex`	`plus(Complex b)`	*sum of this number and b*
`Complex`	`times(Complex b)`	*product of this number and b*
`double`	`abs()`	*magnitude*
`String`	`toString()`	*string representation*

Ex. a = 3 + 4i, b = -2 + 3i.
- a + b = 1 + 7i.
- a × b = -18 + i.
- |a| = 5.

Applications of Complex Numbers

Complex numbers are a quintessential mathematical abstraction.

Applications.
- Fractals.
- Impedance in RLC circuits.
- Signal processing and Fourier analysis.
- Control theory and Laplace transforms.
- Quantum mechanics and Hilbert spaces.
- ...

Complex Number Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args) {
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   Complex c = a.times(b);
   System.out.println("a = " + a);
   System.out.println("b = " + b);
   System.out.println("c = " + c);
}
```

result of `c.toString()`

```
% java TestClient
a = 3.0 + 4.0i
b = -2.0 + 3.0i
c = -24.0 + 1.0i
```

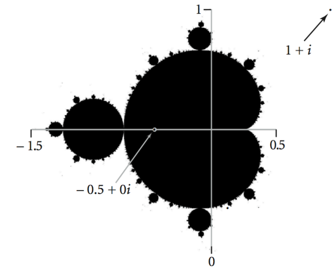
Remark. Can't write `a = b*c` since no operator overloading in Java.

Complex Number Data Type: Implementation

```java
public class Complex {

   private double re;
   private double im;                          // instance variables

   public Complex(double real, double imag) {
      re = real;
      im = imag;
   }                                           // constructor

   public String toString() { return re + " + " + im + "i"; }

   public double abs() { return Math.sqrt(re*re + im*im); }

   public Complex plus(Complex b) {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);   // creates a Complex object, and returns a reference to it
   }

   public Complex times(Complex b) {
      double real = re * b.re – im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }                                           // methods
}
```


Mandelbrot Set

Mandelbrot set. A set of complex numbers.
Plot. Plot (x, y) black if z = x + iy is in the set, and white otherwise.

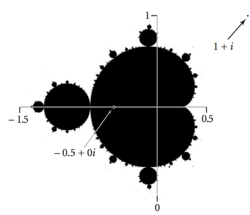


- No simple formula describes which complex numbers are in set.
- Instead, describe using an algorithm.

Mandelbrot Set

Mandelbrot set. Is complex number z_0 is in set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, then z_0 not in set; otherwise z_0 is in set.



t	z_t
0	-1/2 + 0i
1	-1/4 + 0i
2	-7/16 + 0i
3	1+ 0i
4	-79/256 + 0i
5	-26527/65536 + 0i

z = -1/2 is in Mandelbrot set

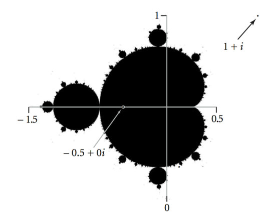
t	z_t
0	1 + i
1	1 + 3i
2	-7 + 7i
3	1 - 97i
4	-9407 – 193i
5	88454401 + 3631103i

z = 1 + i not in Mandelbrot set

Plotting the Mandelbrot Set

Practical issues.

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.



Approximate solution.

- Sample from an N-by-N grid of points in the plane.
- Fact: if $|z_t| \geq 2$ for any t, then z not in Mandelbrot set.
- Pseudo-fact: if $|z_{255}| < 2$ then z "likely" in Mandelbrot set.

Complex Number Data Type: Another Client

Mandelbrot function with complex numbers.

- Is z in the Mandelbrot set?
- Returns white (definitely no) or black (probably yes).
- 7 lines of code with judicious use of data types.

```
public static Color mand(Complex z0) {
   Complex z = z0;
   for (int t = 0; t < 255; t++) {
      if (z.abs() >= 2.0) return Color.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return Color.BLACK;
}
```

$z = z^2 + z_0$

More dramatic picture: replace `Color.WHITE` with grayscale or color.

`new Color(255-t, 255-t, 255-t)`

Complex Number Data Type: Another Client

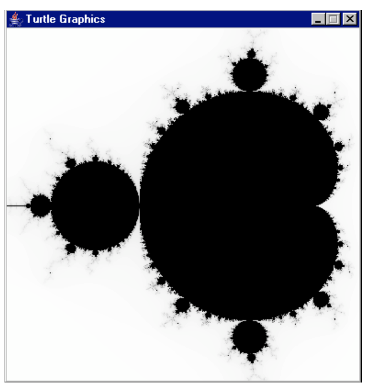
Plot the Mandelbrot set in gray scale.

```
public static void main(String args[]) {
   double xc   = Double.parseDouble(args[0]);
   double yc   = Double.parseDouble(args[1]);
   double size = Double.parseDouble(args[2]);
   int N = 512;
   Picture pic = new Picture(N, N);

   for (int i = 0; i < N; i++) {
      for (int j = 0; j < N; j++) {
         double x = xc - size/2 + size*i/N;
         double y = yc - size/2 + size*i/N;
         Complex z0 = new Complex(x, y);
         Color color = mand(z0);
         pic.set(i, N-1-j, gray);
      }
   }
   pic.show();
}
```

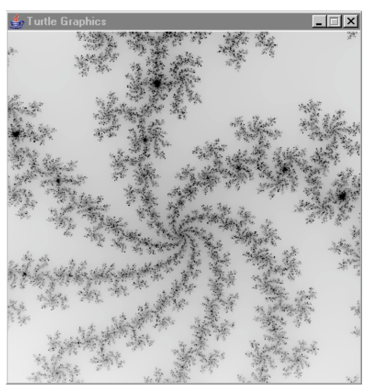
scale to screen coordinates

(0, 0) is upper left

Mandelbrot Set




```
% java Mandelbrot -.5 0 2
```




```
% java Mandelbrot .1045 -.637 .01
```


Vectors

Vector Data Type

Set of values. Sequence of real numbers. [Cartesian coordinates]

API. `public class Vector` (PROGRAM 3.2.4)

	`Vector(double[] a)`	
`Vector`	`plus(Vector b)`	*sum of this vector and b*
`Vector`	`times(double t)`	*scalar product of this vector and t*
`double`	`dot(Vector b)`	*dot product of this vector and b*
`double`	`magnitude()`	*magnitude of this vector*
`Vector`	`direction()`	*unit vector with same direction as this vector*

Ex. $x = (0, 3, 4, 0), y = (0, -3, 1, -4)$.

- $x + y = (0, 0, 5, -4)$.
- $3x = (0, 9, 12, 0)$.
- $x \cdot y = (0 \cdot 0) + (3 \cdot -3) + (4 \cdot 1) + (0 \cdot -4) = -5$.
- $|x| = (0^2 + 3^2 + 4^2 + 0^2)^{1/2} = 5$.
- $\vec{x} = x / |x| = (0, 0.6, 0.8, 0)$.

Vector Data Type Applications

Vectors are a quintessential mathematical abstraction.

Applications.

- Statistics.
- Linear algebra.
- Clustering and similarity search.
- Force, velocity, acceleration, momentum, torque.
- …

Vector Data Type: Implementation

```java
public class Vector {
   private int N;
   private double[] coords;                    // instance variables

   public Vector(double[] a) {
      N = a.length;
      coords = new double[N];
      for (int i = 0; i < N; i++)
         coords[i] = a[i];
   }                                           // constructors

   public double dot(Vector b) {
      double sum = 0.0;
      for (int i = 0; i < N; i++)
         sum += (coords[i] * b.coords[i]);
      return sum;
   }

   public Vector plus(Vector b) {
      double[] c = new double[N];
      for (int i = 0; i < N; i++)
         c[i] = coords[i] + b.coords[i];
      return new Vector(c);                    // methods
   }
```

instance variables

constructors

methods

Vector Data Type: Implementation

```java
   public Vector times(double t) {
      double[] c = new double[N];
      for (int i = 0; i < N; i++)
         c[i] = t * coords[i];
      return new Vector(c);
   }

   public double magnitude() {
      return Math.sqrt(this.dot(this));
   }

   public Vector direction() {
      return this.times(1.0 / this.magnitude());
   }
   ...
```

This. The keyword `this` is a reference to the invoking object.
Ex. When you invoke `a.magnitude()`, `this` is an alias for `a`.

Applications of Data Types

Data type. Set of values and collection of operations on those values.

Simulating the physical world.

- Java objects model real-world objects.
- Ex: point charge, COS 126 student.
- Not always easy to make model reflect reality.

Extending the Java language.

- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Scientific applications: complex, vector, polynomial, matrix, ...