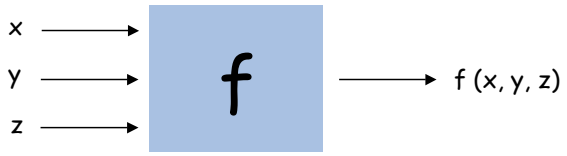
2.1 Functions



Functions (Static Methods)

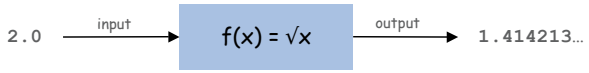
Java function.
- Takes zero or more input arguments.
- Returns one output value.

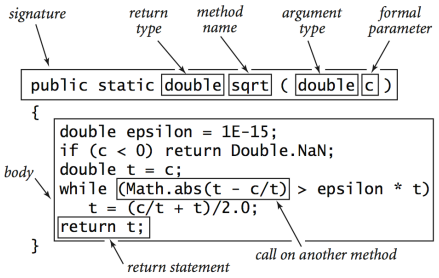
Applications.
- Scientists use mathematical functions to calculate formulas.
- Programmers use functions to build modular programs.
- You use functions for both.

Examples.
- Built-in functions: `Math.random()`, `Math.abs()`, `Integer.parseInt()`.
- User-defined functions: `main()`.

Java Functions

Java functions. Easy to write your own.





Flow of Control

Flow of control. Functions provide a new way to control the flow of execution of a program.

```
public class Newton {

   public static double sqrt(double c)
      // as before

   public static void main(String[] args) {
      double[] a = new double[args.length];
      for (int i = 0; i < args.length; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < a.length; i++)
         System.out.println(Newton.sqrt(a[i]));
   }
}
```

```
% java Newton 1 2 3
1.0
1.414213562373095
1.7320508075688772
```

Scope

Scope. The scope of a variable is the set of statements that can refer to that name.

- Scope of a variable defined within a block is limited to the statements in that block. (including block delimiting function)
- Best practice: declare variables to limit their scope.

```
public class Scope {

   public static int cube(int i) {
      i = i * i * i;
      return i;
   }

   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         System.out.println(i + " " + cube(i));
   }
}
```

two variables named i are independent

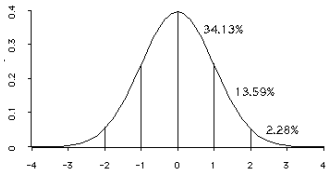
```
% java Scope 3
1 1
2 8
3 27
```


Gaussian Distribution

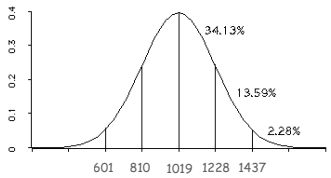
Standard Gaussian distribution.

- "Bell curve."
- Basis of most statistical analysis in social and physical sciences.

Ex. 2000 SAT scores follows a Gaussian distributed with mean μ = 1019, stddev σ = 209.



$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



$$\phi(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} = \phi\left(\frac{x-\mu}{\sigma}\right) / \sigma$$

Java Function for $\phi(x)$

Mathematical functions. Use built-in functions when possible; build your own when not available.

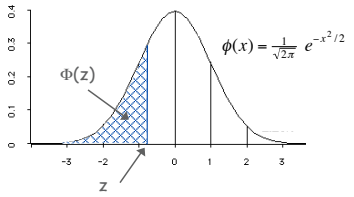
```
public class Gaussian {

   public static double phi(double x) {
      return Math.exp(-x*x / 2) / Math.sqrt(2 * Math.PI);
   }

   public static double phi(double x, double mu, double sigma) {
      return Gaussian.phi((x - mu) / sigma) / sigma;
   }
}
```

Overloading. Functions with different signatures are different.
Multiple arguments. Functions can take any number of arguments.
Calling other functions. Functions can call other functions.

(library or user-defined)

Gaussian Cumulative Distribution Function

Goal. Compute Gaussian cdf $\Phi(z)$.
Challenge. No "closed form" expression and not in Java library.



Taylor series

$$\Phi(z) = \int_{-\infty}^{z} \phi(x)dx = \frac{1}{2} + \phi(z)\left(z + \frac{z^3}{3} + \frac{z^5}{3 \cdot 5} + \frac{z^7}{3 \cdot 5 \cdot 7} + \cdots\right)$$

Bottom line. 1,000 years of mathematical formulas at your fingertips.

Java function for $\Phi(z)$

```
public class Gaussian {

   public static double phi(double x)
      // as before

   public static double Phi(double z) {
      if (z < -8.0) return 0.0;
      if (z >  8.0) return 1.0;
      double sum = 0.0, term = z;
      for (int i = 3; sum + term != sum; i += 2) {
         sum  = sum + term;
         term = term * z * z / i;
      }
      return 0.5 + sum * phi(z);
   }

   public static double Phi(double z, double mu, double sigma) {
      return Phi((z - mu) / sigma);
   }
}
```

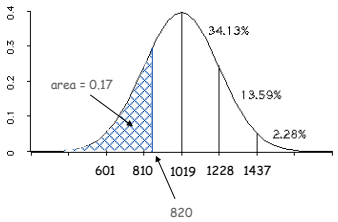
accurate with absolute error less than $8 * 10^{-16}$

$$\Phi(z, \mu, \sigma) = \int_{-\infty}^{z} \phi(z, \mu, \sigma) = \Phi((z - \mu) / \sigma)$$

SAT Scores

Q. NCAA requires at least 820 for Division I athletes. What fraction of test takers in 2000 do not qualify?

A. $\Phi(820, \mu, \sigma) \approx 0.17051$. [approximately 17%]




```
double fraction = Gaussian.Phi(820, 1019, 209);
```


Gaussian Distribution

Q. Why relevant in mathematics?

A. Central limit theorem: under very general conditions, average of a set of variables tends to the Gaussian distribution.

Q. Why relevant in the sciences?

A. Models a wide range of natural phenomena and random processes.

- Weights of humans, heights of trees in a forest.
- SAT scores, investment returns.

Caveat.

> Everybody believes in the exponential law of errors: the experimenters, because they think it can be proved by mathematics; and the mathematicians, because they believe it has been established by observation. - M. Lippman in a letter to H. Poincaré

Building Libraries

Standard Random

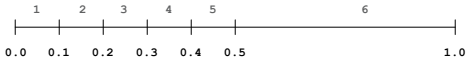
Ex. Library to generate pseudo-random numbers.

```
public class StdRandom {

   public static double uniform(double a, double b) {
      return a + Math.random() * (b-a);
   }

   public static int uniform(int N) {
      return (int) (Math.random() * N);
   }

   public static boolean bernoulli(double p) {
      return Math.random() < p;
   }

   public static double gaussian()
      // recall Assignment 0

   public static double discrete(double[] p)
      // next slide
}
```


Discrete Distribution

Discrete distribution. Given an array of weights (that sum to 1), choose an index at random with probability equal to its weight.

```
p = { 0, .1, .1, .1, .1, .1, .5 }
```




```
public static int discrete(double[] p) {
   // check that weights are nonnegative and sum to 1
   double r = Math.random();
   double sum = 0.0;
   for (int i = 0; i < p.length; i++) {
      sum = sum + p[i];
      if (sum >= r) return i;
   }
   return -1;
}
```

something went wrong

Using a Library

To use the standard random library:

- Put a copy of `StdRandom.java` in current directory.
- Write a client program that uses it.

```
public class LoadedDie {
   public static void main(String args[]) {
      int N = Integer.parseInt(args[0]);
      double[] p = { 0, .1, .1, .1, .1, .1, .5 };
      for (int i = 0; i < N; i++) {
         int die = StdRandom.discrete(p);
         System.out.print(die + " ");
      }
   }
}
```

50% chance of 6,
10% chance of 1-5

```
% javac LoadedDie.java
% java LoadedDie 10
6 5 1 2 6 6 2 6 6 6
```

automatically compiles StdRandom.java if needed

Building Libraries

Functions enable you to build a new layer of abstraction.

- Takes you beyond pre-packaged libraries.
- You build the tools you need: `Math.Phi()`, `StdRandom.uniform()`, …

Process.

- Step 1: identify a useful feature.
- Step 2: implement it.
- Step 3: use it.

- Step 3': re-use it in any of your programs.

Standard Statistics

Ex. Library to compute statistics on an array of real numbers.

```
public class StdStats {

   public static double max(double[] a) {
      double max = Double.NEGATIVE_INFINITY;
      for (int i = 0; i < a.length; i++)
         if (a[i] > max) max = a[i];
      return max;
   }

   public static double mean(double[] a) {
      double sum = 0.0;
      for (int i = 0; i < a.length; i++)
         sum = sum + a[i];
      return sum / a.length;
   }

   public static double stddev(double[] a)
      // see text
```


Modular Programming

Modular Programming

Modular programming.
- Divide program into self-contained pieces.
- Test each piece individually.
- Combine pieces to make program.

Ex. Coupon collector.
- Read parameter from user.
- Choose a random card between 0 and N-1.
- Run one coupon collector simulation.
- Repeat simulation many times.
- Tabulate statistics.
- Print results.

Coupon Collector

Coupon collector function. Given N coupon types, how many coupons do you need to collect until you have at least one of each type?

```
public class Coupon {
   public static int collect(int N) {
      boolean[] found = new boolean[N];
      int cardcnt = 0, valcnt = 0;
      while (valcnt < N) {
         int r = StdRandom.uniform(N);
         if (!found[r]) valcnt++;
         found[r] = true;
         cardcnt++;
      }
      return cardcnt;
   }
}
```

Coupon Collector Experiment

Computational experiment.

- For each N, collect coupons until at least one of each type.
- Repeat experiment several times for each value of N.
- Tabulate statistics and analyze results.

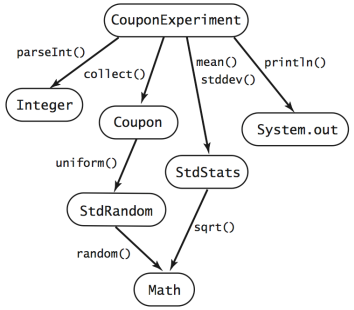
```
public class CouponExperiment {
   public static void main(String[] args) {
      int TRIALS = Integer.parseInt(args[0]);
      double[] results = new double[TRIALS];
      for (int N = 100; N <= 10000; N *= 10) {
         for (int i = 0; i < TRIALS; i++)
            results[i] = Coupon.collect(N);
         double mean   = StdStats.mean(results);
         double stddev = StdStats.stddev(results);
         System.out.printf("%8d %8.2f %8.2f\n", N, mean, stddev);
      }
   }
}
```

formatted printing (ok to ignore details)

```
% java CouponExperiment 10000
   100   516.07   126.10
  1000  7511.94  1300.04
 10000 97778.80 12795.39
```


Coupon Collector: Dependency Graph

Modular programming. Build relatively complicated program by combining several small, independent, modules.



Dependency graph

Functions

Why use functions?

- Makes code easier to understand.
- Makes code easier to debug.
- Makes code easier to maintain.
- Makes code easier to re-use.