



Representation and Matching of Ligand Binding Sites I

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Princeton University
CS597A, Fall 2005



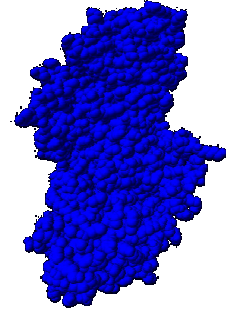
Introduction

Goal:

- Given a protein structure, predict its ligand bindings

Applications:

- Function prediction
- Drug discovery
- etc.



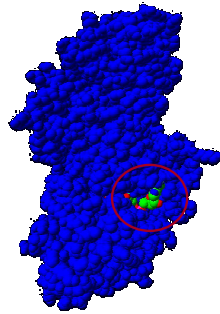
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Introduction

Questions:

- Where will the ligand bind?
- Which ligand will bind?
- How will the ligand bind?
- When?
- Why?
- etc.



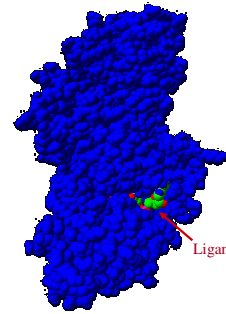
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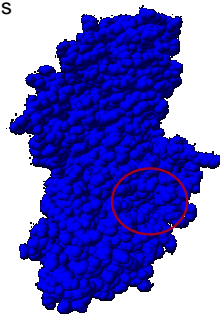


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Which Ligand Will Bind?

Possible matching strategies

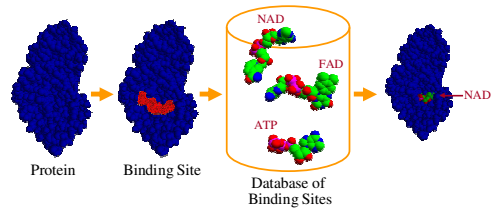


Which Ligand Will Bind?

Possible matching strategies

• Binding site → Ligands

Protein-Ligand Docking
(after fall break)



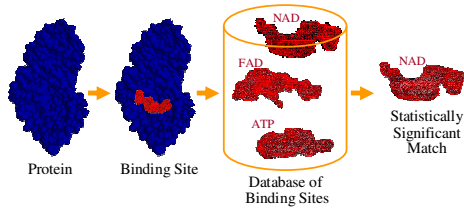
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Possible matching strategies

- Binding site → Ligands
- ∅ Binding site → Binding sites

Binding Site Matching
(next few lectures)



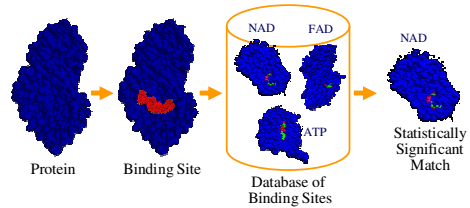
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Possible matching strategies

- Binding site → Ligands
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- ∅ Binding site → Proteins

Binding Site Search
(next few lectures)

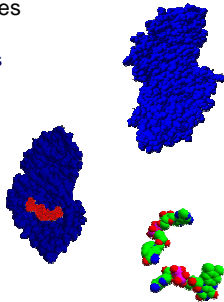


Which Ligand Will Bind?



Possible matching strategies

- Binding site → Ligands
- Binding site → Binding sites
- Binding site → Proteins
- Protein → Ligands
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- Ligand → Ligands
- Ligand → Binding sites
- Ligand → Proteins

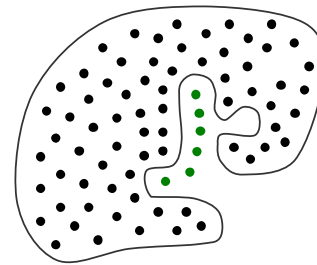


Binding Site Representation



Possible descriptions:

- Point set
- Surface
- Volume

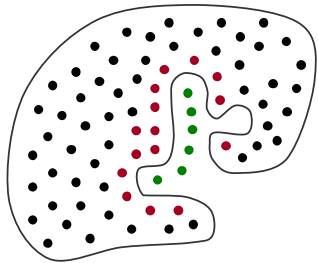


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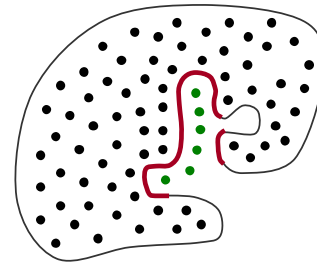


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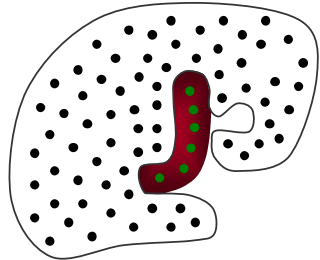


Binding Site Representation



Possible descriptions:

- Point set
- Surface
- Volume



Course Schedule



Next few lectures

- 10/10: Binding site point sets
- 10/12: Discussion with Helen Berman
- 10/17: Binding site templates
- 10/19: Project proposals
- 10/24: Binding site surfaces
- 10/26: Binding site volumes

After fall break

- Protein-ligand docking
- Protein-protein docking
- Drug screening and design
- Structure determination

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Outline



Introduction

Point set representations ←

Point set matching

- Association graphs
- Geometric hashing
- Iterative closest point

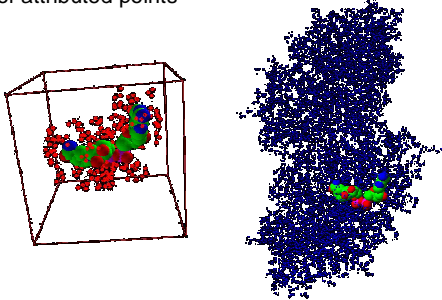
Evaluation

Discussion

Point Set Representation



Set of attributed points



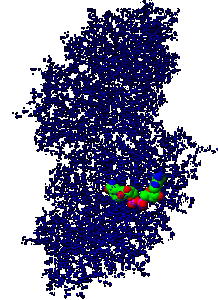
[hhd]

Point Set Representation



Set of attributed points

- Atoms
- Residues
- Pseudo-centers
- Surface critical points
- etc.



[hhd]

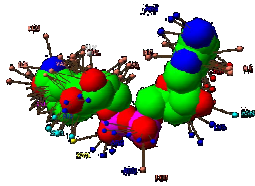
Point Set Representation



Set of attributed points

∅ Atoms

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- etc.



Key Atoms Surrounding Binding Site

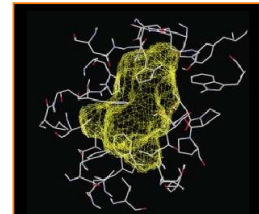
[hhd]

Point Set Representation



Set of attributed points

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- etc.



Residues Surrounding Binding Site

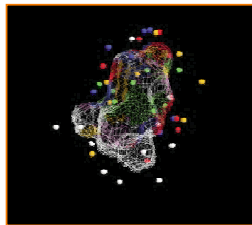
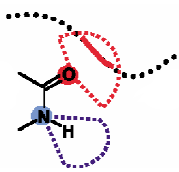
[Schmitt02]

Point Set Representation



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Residues Surrounding Binding Site

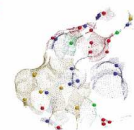
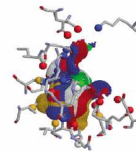
[Schmitt02]

Point Set Representation



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- etc.



Surface Curvature

Represent Chemical and Geometric Properties of Surface

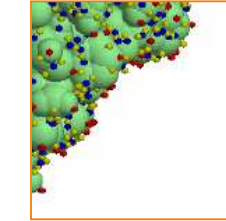
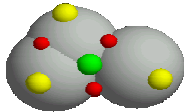
[Shulman-Peleg04]

Point Set Representation



Set of attributed points

- Atoms
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- Pseudo-centers
- Surface critical points
- etc.



Critical Points on Surface of Binding Site

[Lin94] [Wolfson]

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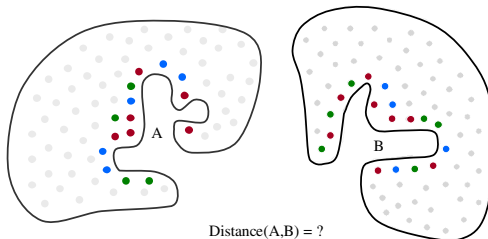
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Point Set Matching



Goal is to compute a distance measure for a pair of attributed point sets

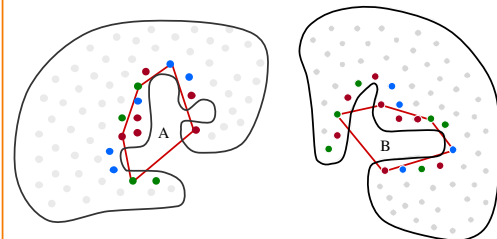


Point Set Matching



Challenge is to find corresponding points

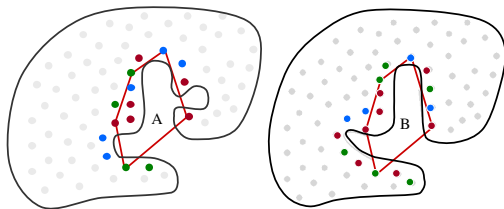
- "Subset of points in A" may match "subset of points in B"



Point Set Matching



Calculating a superposition and distance measure is easy if correspondences are known (proposed)



Point Set Matching



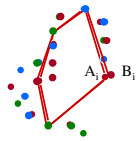
Calculating a superposition and distance measure is easy if correspondences are known (proposed)



Point Set Matching



Calculating a superposition and distance measure is easy if correspondences are known (proposed)



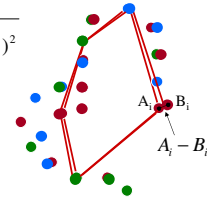
Least-squares optimal superposition of corresponding points

Point Set Matching



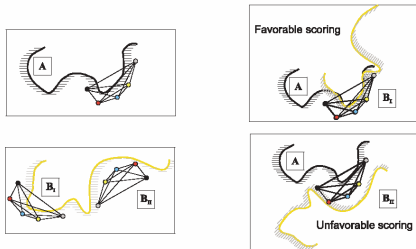
Calculating a superposition and distance measure is easy if correspondences are known (proposed)

$$RMSD(A, B) = \sqrt{\sum_{i=1}^N (A_i - B_i)^2}$$



Distance(A,B) = RMSD(A,B) + OtherTerms ...

Point Set Match Scoring



Aligned points may be examined further to compute scoring function (e.g., check if surfaces align)

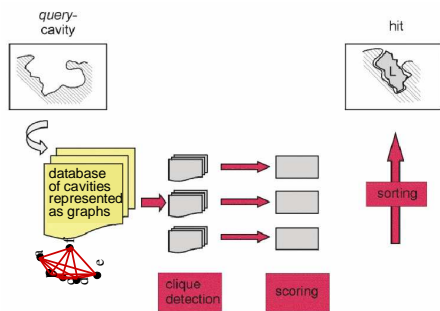
[Schmitt02]

Outline



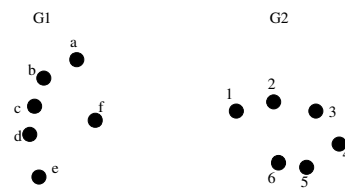
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Matching with Association Graphs



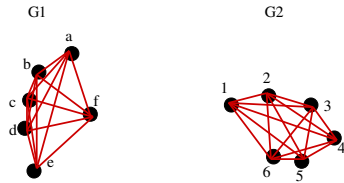
[Schmitt02]

Graph Representation



[Schmitt02, Brown82]

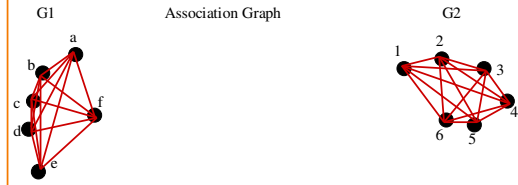
Graph Representation



Represent both points sets as complete graphs (G1 and G2).
(edges connect all pairs of vertices within each point set)

[Schmitt02, Brown82]

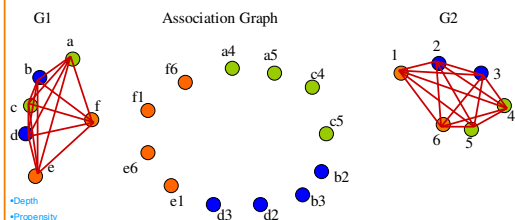
Association Graph



Create vertices in the association graph for all compatible pairs of vertices in the original graphs. This can lead to a large number of vertices.

[Schmitt02, Brown82]

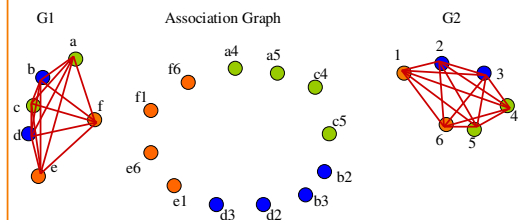
Association Graph



Create vertices in the association graph for all compatible pairs of vertices in the original graphs. Compatibility could refer to chemical properties.

[Schmitt02, Brown82]

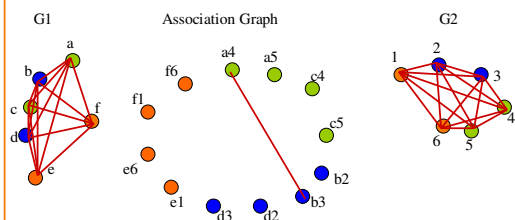
Association Graph



Create edges between (uv) and (wx) if the edges between (u) and (w) as well as between (v) and (x) match.

[Schmitt02, Brown82]

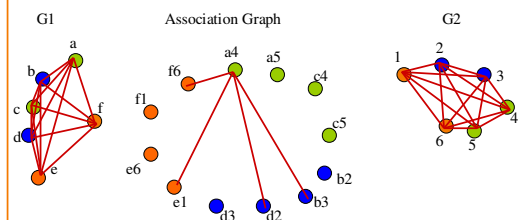
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Create edges between (uv) and (wx) if the edges between (u) and (w) as well as between (v) and (x) match. For this example, edge length is the only consideration

[Schmitt02, Brown82]

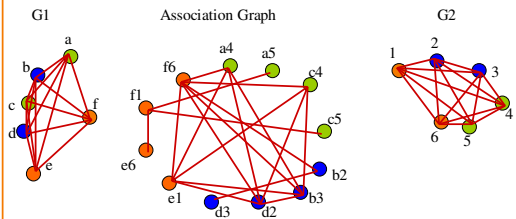
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[Schmitt02, Brown82]

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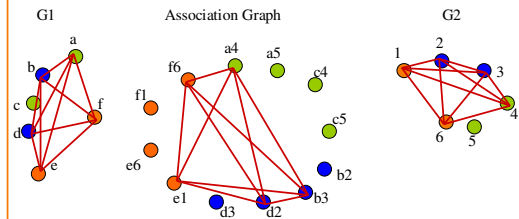


Create edges between (uv) and (wx) if the edges between (u) and (w) as well as between (v) and (x) match.

For this example, edge length is the only consideration

[Schmitt02, Brown82]

Finding Correspondences



The largest set of corresponding nodes in the same configuration is the **maximal clique** in the association graph (i.e., the largest subset of the association graph for which all nodes are connected to one another).

[Schmitt02, Brown82]

Finding Correspondences



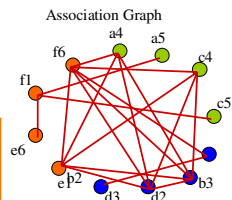
Computational complexity:

- $O(2^n)$ for n points
- NP-complete

```

Find the Maximal Clique{
  return Cliques(empty, all nodes)
}

Cliques(X, Y){
  if (no node in Y-X is connected to all of X){
    return X;
  } else {
    y = node in Y connected to all of X;
    return Largest(Cliques(X union y, Y),
                  Cliques(X, Y-y));
  }
}
    
```



[Schmitt02, Brown82]

Outline

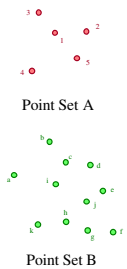


- Introduction
- Point set representations
- Point set matching
 - Association graphs
 - ~~Geometric hashing~~
 - Iterative closest point
- Evaluation
- Discussion

Geometric Hashing



Discretize transformations and scoring

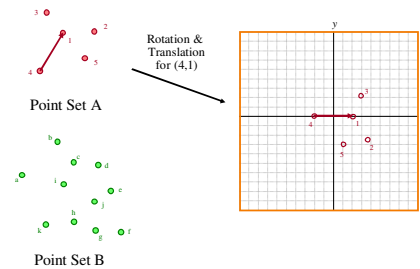


[Wolfson97]

Geometric Hashing



Discretize transformations and scoring

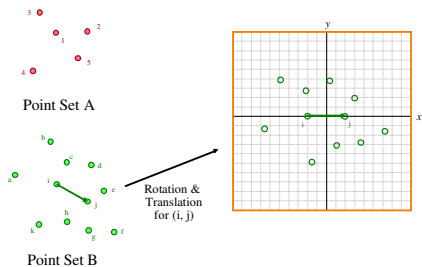


[Wolfson97]

Geometric Hashing



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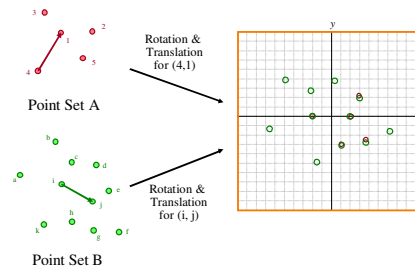


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Geometric Hashing



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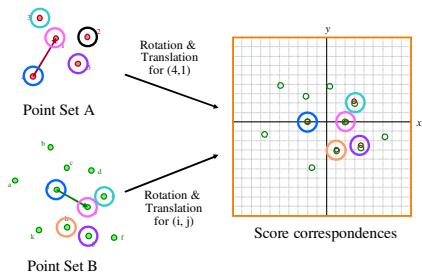


[Wolfson97]

Geometric Hashing



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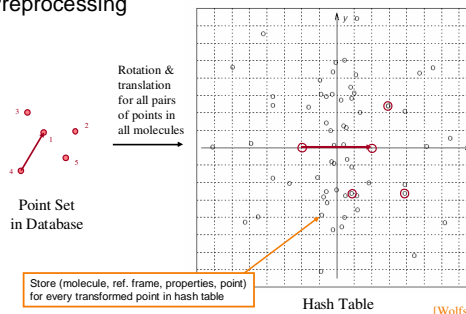


[Wolfson97]

Geometric Hashing



Preprocessing

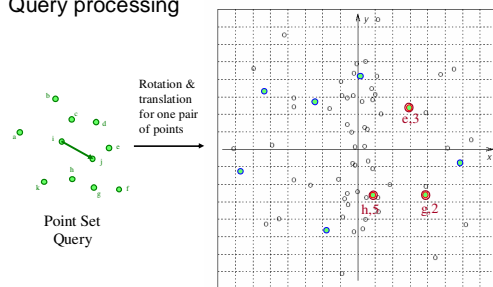


[Wolfson97]

Geometric Hashing



Query processing



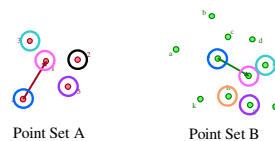
[Wolfson97]

Geometric Hashing



Further processing

- Refine alignment based on computed correspondences

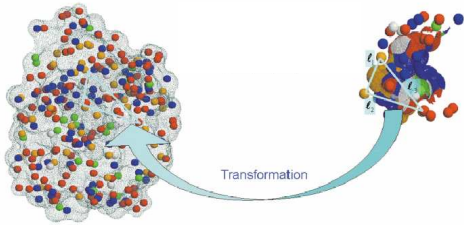


[Wolfson97]

Geometric Hashing



Create transformations for triples of points in 3D



[Shulman-Peleg04]

Geometric Hashing



Preprocessing

- For each triple of points
- Compute reference frame
- For each point
- Transform point into reference frame
- Hash (molecule, ref. frame, properties, point)

Query processing

- Choose any triple of points
- Compute reference frame
- For each point
- Transform point into reference frame
- For each entry in hash bin for transformed point
- Check point properties
- Vote for (molecule, ref. frame)

Geometric Hashing



Preprocessing complexity

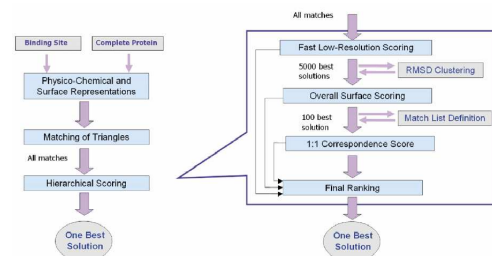
- $O(n^4)$ for n points per binding site
- § $O(n^3)$ possible triples * $O(n)$ transformations per triple

Query complexity

- $O(m)$ * binsize for m points in query binding site
- § 1 triple * $O(m)$ transformations per triple * binsize hash processing per transformation

[Wolfson97]

Shulman-Peleg et al. 2004



[Shulman-Peleg04]

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- Iterative closest point

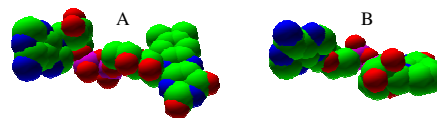
Evaluation

Discussion

Iterative Closest Point



Given two molecules

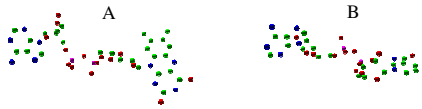


[Bes192]

Iterative Closest Point



Given two molecules

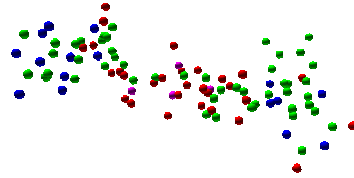


[Bes192]

Iterative Closest Point



Given two molecules and an initial guess for the transformation that aligns them

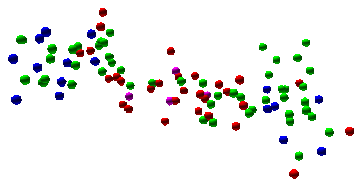


[Bes192]

Iterative Closest Point



Assume closest points correspond

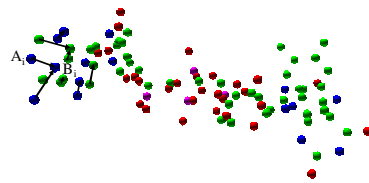


[Bes192]

Iterative Closest Point



Assume closest points correspond: $A \rightarrow B$

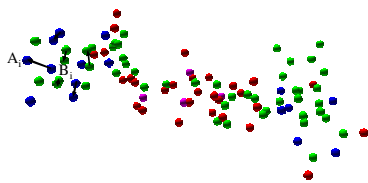


[Bes192]

Iterative Closest Point



Assume closest points correspond: $A \rightarrow B$ and $B \rightarrow A$

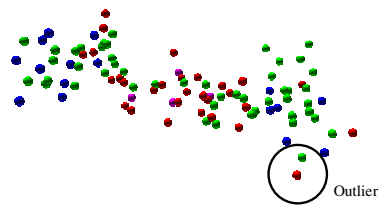


[Bes192]

Iterative Closest Point



Rejecting outliers

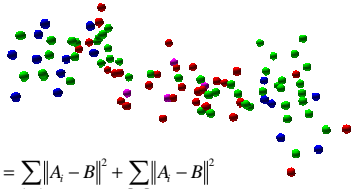


[Bes192]

Iterative Closest Point



Find the transformation that optimally aligns proposed correspondences (superposition)



$$d(A, B) = \sum_{A_i \in A} \|A_i - B\|^2 + \sum_{B_j \in B} \|A_j - B\|^2$$

[Bes192]

Iterative Closest Point



Iterate until convergence

1. Select source points (from one or both molecules)
2. Match to points in the other molecule
3. Weight the correspondences
4. Reject outlier point pairs
5. Compute an error metric for the current transform
6. Minimize the error metric w.r.t. transformation

Computational complexity

- $O(k * n \log n)$ for n points per binding site and k iterations
- § k iterations * $O(n)$ points * $O(\log n)$ to find closest point

Slide courtesy of Szymon Rusinkiewicz

Iterative Closest Point



Demo

Demo courtesy of Szymon Rusinkiewicz

Summary



Association graphs

- Expensive for large point sets
- Distance threshold for "associations"

Geometric hashing

- Fast query, after slow preprocessing
- Distance threshold implicit in hash bucket sizes

Iterative closest points

- Fast, in practice
- Allows soft scoring functions
- Requires good initial guess

Outline



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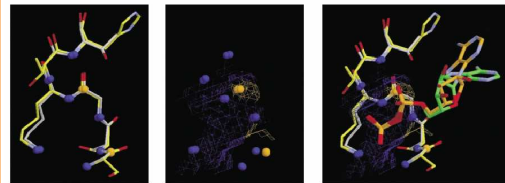
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- Association graphs
- Geometric hashing
- Iterative closest point

Evaluation ←

Discussion

Aligned Points



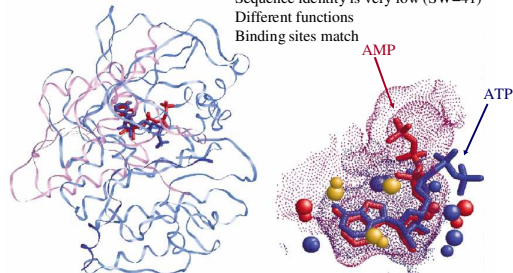
Equivalent phosphate binding areas in the binding pockets of uridylyl kinase (lukz) and the structure of a kinesine-type domain (3kar).

Sequence identity is very low (SW=41)

Different functions
Binding sites match

[Schmitt02]

Aligned Points



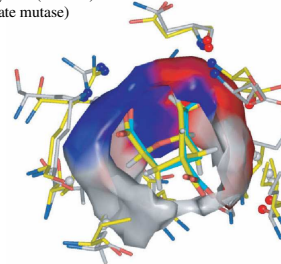
Statistical significance?

[Shulman-Peleg04]

Aligned Points



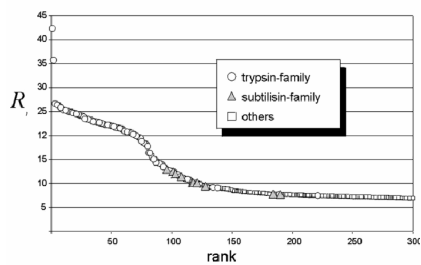
Sequence identity is very low (SW=42)
Same function (chorismate mutase)
Binding sites match



Superposition of the binding pockets from the chorismate mutases 1csm and 4csm

[Weskamp04]

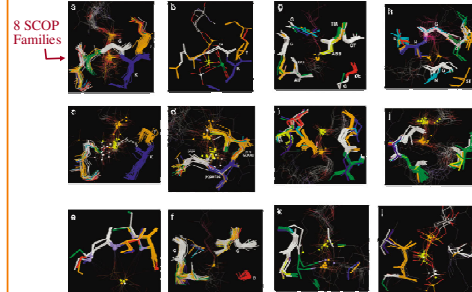
Ranked Matches



Results of query with binding site of trypsin structure (1tpo)

[Schmitt02]

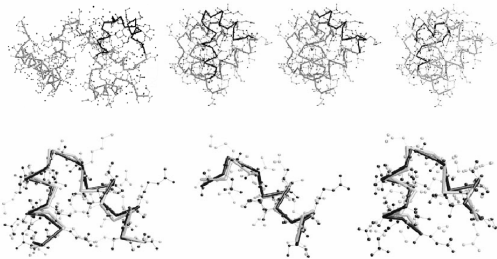
Clusters of Point Sets



Clusters of 476 phosphate binding sites

[Brakoulias04]

Detected Structural Motifs



[Pennec98]

Generally Speaking ...



Small sets of proteins

- Serine proteases (catalytic triad)
- Adenine-binding proteins (largest source of data)

Focus on true positives

- False positives, false negatives?
- Aggregate statistics for large set of queries?
- Statistical significance? [Stark03]

Rarely provide comparison to related approaches

- Comparison to sequence-based matching methods?
- Comparison to other local structure matching methods?

Discussion



?

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