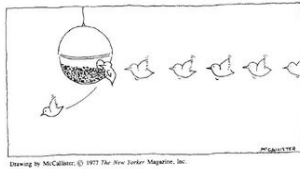


## 2.5 Stacks and Queues



Robert Sedgewick and Kevin Wayne · Copyright © 2005 · <http://www.Princeton.EDU/~cos226>

### Client, Implementation, Interface

Separate interface and implementation so as to:

- Build layers of abstraction.
- Reuse software.
- Ex: stack, queue, symbol table.

**Interface:** description of data type, basic operations.  
**Client:** program using operations defined in interface.  
**Implementation:** actual code implementing operations.

Fundamental data types.

- Set of operations (**add**, **remove**, **test if empty**) on generic data.
- Intent is clear when we insert.
- Which item do we remove?

**Stack.**

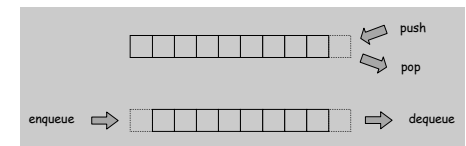
- Remove the item **most recently added**.
- Analogy: cafeteria trays, Web surfing.

LIFO = "last in first out"

**Queue.**

- Remove the item **least recently added**.
- Analogy: Registrar's line.

FIFO = "first in first out"



### Client, Implementation, Interface

**Benefits.**

- Client can't know details of implementation, so has many implementation from which to choose.
- Implementation can't know details of client needs, so many clients can re-use the same implementation.
- **Design:** creates modular, re-usable libraries.
- **Performance:** use optimized implementation where it matters.

**Interface:** description of data type, basic operations.  
**Client:** program using operations defined in interface.  
**Implementation:** actual code implementing operations.

## Stack

### Stack operations.

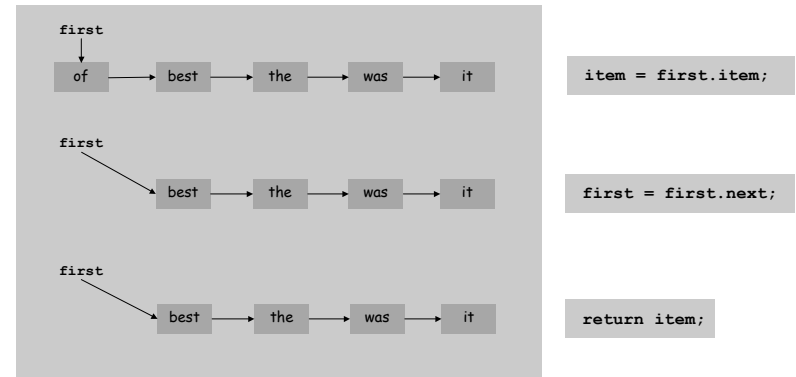
- `push()`      Insert a new item onto stack.
- `pop()`        Delete and return the item most recently added.
- `isEmpty()`    Is the stack empty?



```
public static void main(String[] args) {
    StringStack stack = new StringStack();
    while(!StdIn.isEmpty()) {
        String s = StdIn.readString();
        stack.push(s);
    }
    while(!stack.isEmpty()) {
        String s = stack.pop();
        System.out.println(s);
    }
}
// a sample stack client
```

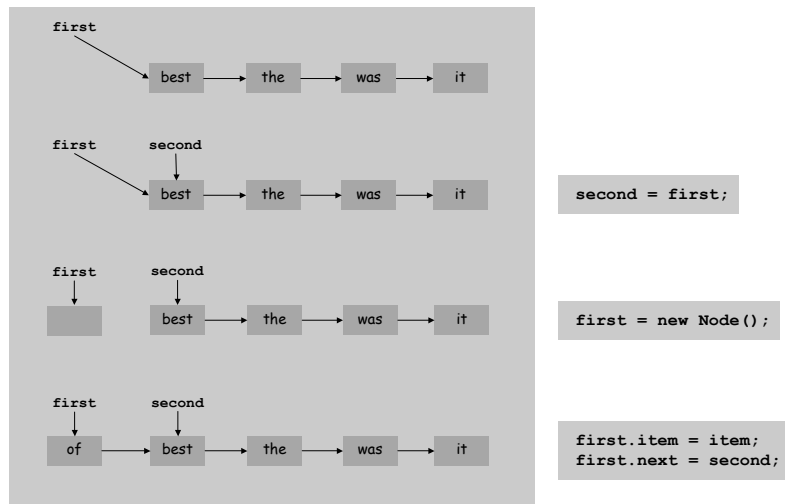
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## Stack Pop: Linked List Implementation



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## Stack Push: Linked List Implementation



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## Stack: Linked List Implementation

```
public class StringStack {
    private Node first = null;

    private class Node {
        String item;
        Node next;
    }

    public boolean isEmpty() { return first == null; }

    public void push(String item) {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

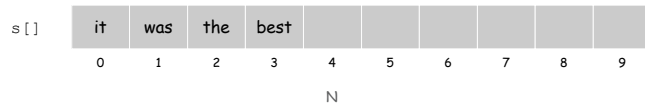
    public String pop() {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

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## Stack: Array Implementation

### Array implementation of a stack.

- Use array `s` to store `N` items on stack.
- `push()` add new item at `s[N]`.
- `pop()` remove item from `s[N-1]`.



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## Stack: Array Implementation

```
public class StringStack {
    private String[] s;
    private int N = 0;

    public StringStack(int capacity) {
        s = new String[capacity];
    }

    public boolean isEmpty() { return N == 0; }

    public void push(String item) {
        s[N++] = item;
    }

    public String pop() {
        String item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }
}
```

garbage collector only reclaims memory if no outstanding references

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## Stack Array Implementation: Resizing

How to resize array? Increase size `s` by one if the array is full.

### Thrashing.

- Increasing the size of an array involves copying all of the elements to a new array.
- Inserting `N` elements: time proportional to  $1 + 2 + \dots + N \approx N^2/2$ .  
 $N = 1 \text{ million} \Rightarrow$  infeasible.

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## Stack Array Implementation: Dynamic Resizing

How to resize array? Use **repeated doubling**: if `s` not big enough, create new array of twice the size, and copy items.

```
public StringStack() { this(8); }
public void push(String item) {
    if (N >= s.length) resize();
    s[N++] = item;
}
private void resize() {
    String[] dup = new String[2*N];
    for (int i = 0; i < N; i++)
        dup[i] = s[i];
    s = dup;
}
```

no-argument constructor

double the size of the array

**Consequence.** Inserting `N` items takes time proportional to  $N$  (not  $N^2$ ).

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## Stack Implementations: Array vs. Linked List

**Stack implementation tradeoffs.** Can implement with either array or linked list, and client can use interchangeably. Which is better?

### Array.

- Most operations take constant time.
- Expensive re-doubling operation every once in a while.
- Any sequence of N operations (starting from empty stack) takes time proportional to N.

← "amortized" bound

### Linked list.

- Grows and shrinks gracefully.
- Every operation takes constant time.
- Uses extra space and time to deal with references.

## Queue

### Queue operations.

- enqueue () Insert a new item onto queue.
- dequeue () Delete and return the item least recently added.
- isEmpty () Is the queue empty?

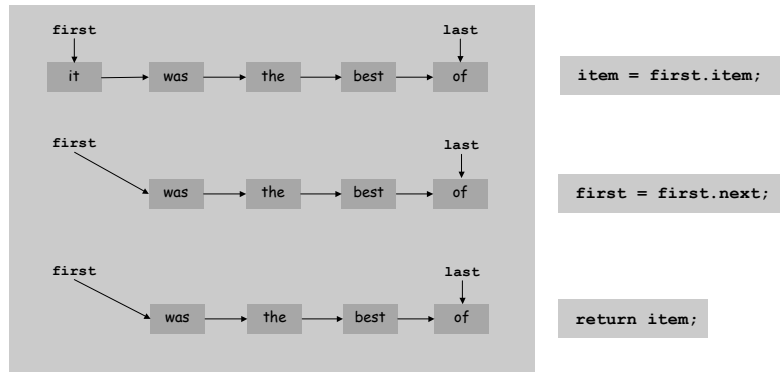
```
public static void main(String[] args) {
    StringQueue q = new StringQueue();
    q.enqueue("Vertigo");
    q.enqueue("Just Lose It");
    q.enqueue("Pieces of Me");
    q.enqueue("Pieces of Me");
    System.out.println(q.dequeue());
    q.enqueue("Drop It Like It's Hot");
    while(!q.isEmpty())
        System.out.println(q.dequeue());
}
```



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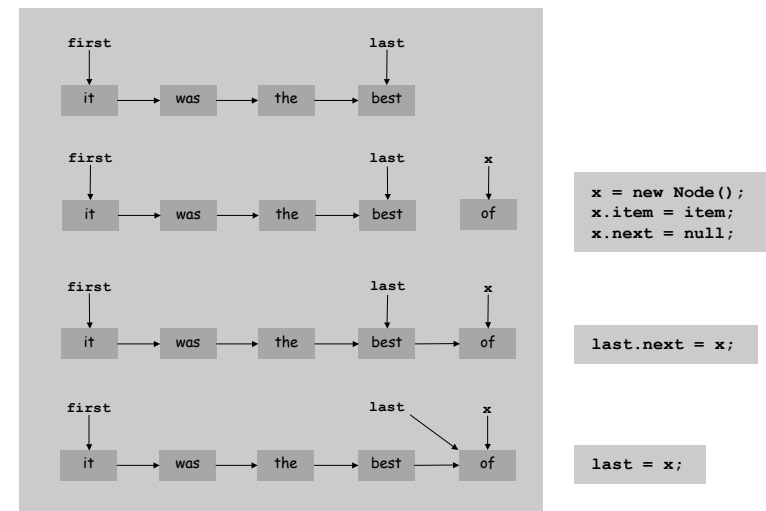
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## Dequeue: Linked List Implementation



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## Enqueue: Linked List Implementation



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## Queue: Linked List Implementation

```
public class StringQueue {
    private Node first;
    private Node last;

    private class Node { String item; Node next; }

    public boolean isEmpty() { return first == null; }

    public void enqueue(String item) {
        Node x = new Node();
        x.item = item;
        x.next = null;
        if (isEmpty()) { first = x; last = x; }
        else { last.next = x; last = x; }
    }

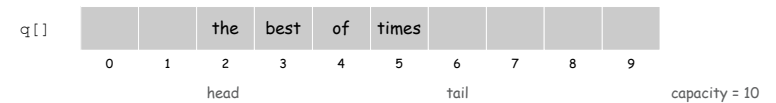
    public String dequeue() {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

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## Queue: Array Implementation

### Array implementation of a queue.

- Use array `q` to store items on queue.
- `enqueue()`: add new object at `q[tail]`.
- `dequeue()`: remove object from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.



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## Parameterized Data Types

### Parameterized Data Types

We implemented: `StringStack`, `StringQueue`.

We also want: `URLStack`, `CustomerQueue`, etc?

**Attempt 1.** Implement a separate stack class for each type.

- Rewriting code is tedious and **error-prone**.
- Maintaining cut-and-pasted code is tedious and **error-prone**.

## Stack of Objects

We implemented: `StringStack`, `StringQueue`.

We also want: `URLStack`, `CustomerQueue`, etc?

Attempt 2. Implement a stack with items of type `Object`.

- Casting is required in client.
- Casting is error-prone: **run-time error** if types mismatch.

```
Stack s = new Stack();
Apple a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

## Generics

Generics. [since Java 1.5] Parameterize stack by a single type.

- Avoid casting in both client and implementation.
- Discover type mismatch errors at **compile-time** instead of run-time.

```
Stack<Apple> s = new Stack<Apple>();
Apple a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = s.pop();
```

parameter

compile-time error

no cast needed in client

Guiding principle. Run-time errors are much harder to identify and fix than compile-time errors.

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## Generic Stack: Linked List Implementation

```
public class Stack<Item> {
    private Node first;

    private class Node {
        Item item;
        Node next;
    }

    public boolean isEmpty() { return first == null; }

    public void push(Item item) {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

    public Item pop() {
        Item item = first.item;
        first = first.next;
        return item;
    }
}
```

arbitrary parameterized type name, but must use consistently

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## Generic Stack: Array Implementation

The way it should be.

```
public class ArrayStack<Item> {
    private Item[] a;
    private int N;

    public ArrayStack(int capacity) {
        a = new Item[capacity];
    }

    public boolean isEmpty() { return N == 0; }

    public void push(Item item) {
        a[N++] = item;
    }

    public Item pop() {
        return a[--N];
    }
}
```

@#\*\$! generic array creation not allowed in Java

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The way it is: an **ugly cast** in the implementation.

```
public class ArrayStack<Item> {
    private Object[] a;
    private int N;

    public ArrayStack(int capacity) {
        a = new Object[capacity];
    }

    public boolean isEmpty() { return N == 0; }

    public void push(Item item) {
        a[N++] = item;
    }

    public Item pop() {
        return (Item) a[--N];
    }
}
```

↖ the ugly cast

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## Stacks and Queues: Applications

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**Generic stack implementation.** Allows objects, not primitive types.

**Wrapper type.**

- Each primitive type has a **wrapper** object type.
- Ex: Integer is wrapper type for int.

**Autoboxing.** Automatic cast between a primitive type and its wrapper.

**Syntactic sugar.** Casts are still done behind the scenes.

```
Stack<Integer> s = new Stack<Integer>();
s.push(17);      // s.push(new Integer(17));
int a = s.pop(); // int a = ((Integer) s.pop()).intValue();
```

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## Stack Applications

**Real world applications.**

- Parsing in a compiler.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

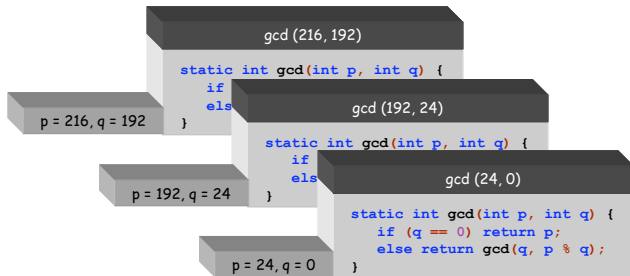
## Function Calls

How a compiler implements functions.

- Function call: **push** local environment and return address.
- Return: **pop** return address and local environment.

**Recursive function.** Function that calls itself.

**Note.** Can always use a stack to remove recursion.



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## Postfix Notation

Postfix notation.

- Put operator after operands in expression.
- Use stack to evaluate.
  - operand: push it onto stack
  - operator: pop operands, push result
- Systematic way to save intermediate results and avoid parentheses.



```

% java Postfix
1 2 3 4 5 * + 6 * * +
277    infix expression: (1+((2*((3+(4*5))*6)))

% java Postfix
7 16 * 5 + 16 * 3 + 16 * 1 +
30001 convert 7531 from hex to decimal using Horner's method
    
```



J. Lukasiewicz

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## Postfix Evaluation

```

public class Postfix {
  public static void main(String[] args) {
    Stack<Integer> st = new Stack<Integer>();
    while (!StdIn.isEmpty()) {
      String string = StdIn.readString();
      if (s.equals("+")) st.push(st.pop() + st.pop());
      else if (s.equals("*")) st.push(st.pop() * st.pop());
      else st.push(Integer.parseInt(s));
    }
    System.out.println(st.pop());
  }
}
    
```

```

% java Postfix
1 2 3 4 5 * + 6 * * +
277

% java Postfix
7 16 * 5 + 16 * 3 + 16 * 1 +
30001
    
```

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## Infix to Postfix

Infix to postfix algorithm.

- Left parentheses: ignore.
- Right parentheses: pop and print.
- Operator: push.
- Integer: print.

```

% java Infix
( 2 + ( ( 3 + 4 ) * ( 5 * 6 ) ) )
* 2 3 4 + 5 6 * * +

% java Infix | java Postfix
( 2 + ( ( 3 + 4 ) * ( 5 * 6 ) ) )
212
    
```

```

public class Infix {
  public static void main(String[] args) {
    Stack<String> stack = new Stack<String>();
    while (!StdIn.isEmpty()) {
      String s = StdIn.readString();
      if (s.equals("(")) System.out.print(stack.pop() + " ");
      else if (s.equals("(")) System.out.print("(");
      else if (s.equals("+")) stack.push(s);
      else if (s.equals("*")) stack.push(s);
      else System.out.print(s + " ");
    }
  }
}
    
```

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## Queue Applications

### Some applications.

- iTunes playlist.
- Breadth first search.
- Data buffers (iPod, TiVo).
- Graph processing (stay tuned).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

### Simulations of the real world.

- Traffic analysis of Lincoln tunnel.
- Waiting times of customers in McDonalds.
- Determining number of cashiers to have at a supermarket.

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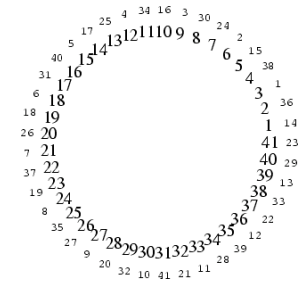
## Josephus Problem

### Flavius Josephus. [first century]

- Band of 41 Jewish rebels trapped in cave by Romans.
- Preferring suicide to capture, rebels formed a circle and killed every 3<sup>rd</sup> remaining person until no one was left.

Q. Where should you stand to be the last survivors?

A. 31.

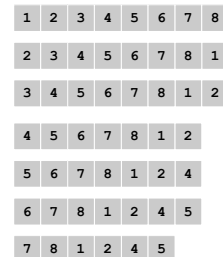


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## Josephus Problem

```
public class Josephus {
    public static void main(String[] args) {
        int M = Integer.parseInt(args[0]);
        int N = Integer.parseInt(args[1]);
        Queue<Integer> q = new Queue<Integer>();
        for (int i = 1; i <= N; i++)
            q.enqueue(i);
        while (!q.isEmpty()) {
            for (int i = 0; i < M - 1; i++)
                q.enqueue(q.dequeue());
            System.out.print(q.dequeue() + " ");
        }
    }
}
```

N = 8, M = 3



```
% java Josephus 5 9
5 1 7 4 3 6 9 2 8
```

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## M/M/1 Queuing Model

### M/M/1 queue.

- Customers arrive at rate of  $\lambda$  per minute.
- Customers are serviced at rate of  $\mu$  per minute.
- Inter-arrival time obeys exponential distribution:  $\Pr[X \leq x] = 1 - e^{-\lambda x}$



Q. How long does a customer wait in queue?

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## M/M/1 Queue: Implementation

```
public class MM1Queue {
    public static void main(String[] args) {
        double lambda = Double.parseDouble(args[0]);
        double mu = Double.parseDouble(args[1]);
        Queue<Double> q = new Queue<Double>();
        double nextArrival = Random.exponential(lambda);
        double nextDeparture = Random.exponential(mu);
        while(true) {
            if (nextArrival < nextDeparture) {           arrival
                q.enqueue(nextArrival);
                nextArrival += StdRandom.exp(lambda);
            }
            else {                                       departure
                if (!q.isEmpty()) {
                    double wait = nextDeparture - q.dequeue();
                    // add waiting time to histogram
                }
                nextDeparture += Random.exponential(mu);
            }
        }
    }
}
```

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### Summary

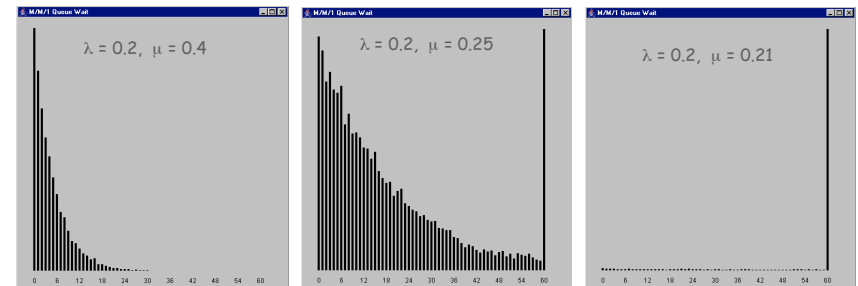
Stacks and queues are fundamental ADTs.

- Array implementation.
- Linked list implementation.
- Different performance characteristics.

Many applications.

## M/M/1 Queue Analysis

Remark. As service rate approaches arrival rate, service goes to  $h^{***}$ .



Theorem. Average time a customer spends in system =  $1 / (\mu - \lambda)$ .

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### Summary

ADTs enable modular programming.

- Separate compilation.
- Split program into smaller modules.
- Different clients can share the same ADT.

ADTs enable encapsulation.

- Keep modules independent (include `main` in class for testing).
- Can substitute different classes that implement same interface.
- No need to change client.

Issues of ADT design.

- Feature creep.
- Formal specification problem.
- Implementation obsolescence.

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