7. Theory of Computation

Introduction to Theoretical CS

Two fundamental questions.
- What can a computer do?
- What can a computer do with limited resources?

General approach.
- Don't talk about specific machines or problems.
- Consider minimal abstract machines.
- Consider general classes of problems.

Pentium IV running Linux kernel 2.4.22

Why Learn Theory

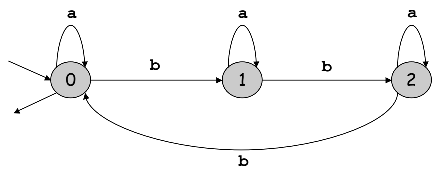
In theory …
- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice …
- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

"In theory there is no difference between theory and practice. In practice there is." -Yogi Berra

Regular Expressions and DFAs

`a* | (a*ba*ba*ba*)*`



Pattern Matching Applications

Test if a string matches some pattern.

- Process natural language.
- Scan for virus signatures.
- Search for information using Google.
- Access information in digital libraries.
- Retrieve information from Lexis/Nexis.
- Search-and-replace in a word processors.
- Filter text (spam, NetNanny, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).
- Search for markers in human genome using PROSITE patterns.

Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read in data stored in TOY input file format.
- Automatically create Java documentation from Javadoc comments.

Regular Expressions: Basic Operations

Regular expression. Notation to specify a set of strings.

Operation	Regular Expression	Yes	No
Concatenation	`aabaab`	`aabaab`	every other string
Wildcard	`.u.u.u.`	`cumulus` `jugulum`	`succubus` `tumultuous`
Union	`aa | baab`	`aa` `baab`	every other string
Closure	`ab*a`	`aa` `abbba`	`ab` `ababa`
Parentheses	`a(a|b)aab`	`aaaab` `abaab`	every other string
	`(ab)*a`	`a` `ababababa`	ε `abbbaa`

Regular Expressions: Examples

Regular expression. Notation is surprisingly expressive.

Regular Expression	Yes	No
`.* spb .*` contains the trigraph `spb`	`raspberry` `crispbread`	`subspace` `subspecies`

`a* | (a*ba*ba*ba*)*`
multiple of three `b`'s
	`bbb`
	`aaa`
	`bbbaababbaa`
		`b`
		`bb`
		`baabbbaa`

`.*0....`
fifth to last digits is `0`
	`1000234`
	`98701234`
		`111111111`
		`403982772`

`gcg (cgg|agg)* ctg`
fragile X syndrome indicator
	`gcgctg`
	`gcgcggctg`
	`gcgcggaggctg`
		`gcgcgg`
		`cggcggcggctg`
		`gcgcaggctg`

Generalized Regular Expressions

Regular expressions are a standard programmer's tool.

- Built in to Java, Perl, Unix, Python, ...
- Additional operations typically added for convenience.
- Ex: `[a-e]+` is shorthand for `(a|b|c|d|e)(a|b|c|d|e)*`.

Operation	Regular Expression	Yes	No
One or more	`a(bc)+de`	`abcde` `abcbcde`	`ade` `bcde`

Character classes	`[A-Za-z][a-z]*`	`capitalized`
		`Word`
			`camelCase`
			`4illegal`

Exactly k	`[0-9]{5}-[0-9]{4}`	`08540-1321`
		`19072-5541`
			`111111111`
			`166-54-111`

Negations	`[^aeiou]{6}`	`rhythm`	`decade`

Regular Expressions in Java

Validity checking. Is `input` in the set described by the `re`?

```
public class Validate {
   public static void main(String[] args) {
      String re    = args[0];
      String input = args[1];
      System.out.println(input.matches(re));
   }
}
```

powerful string library method

```
% java RE "..oo..oo." bloodroot
true

% java RE "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true

% java RE "[a-z]+@([a-z]+\.)+(edu|com)" rs@cs.princeton.edu
true
```

need help solving crosswords?

legal Java identifier

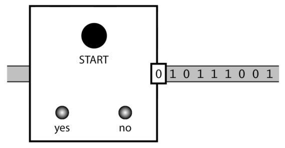
valid email address (simplified)

need quotes to "escape" the shell

Solving the Pattern Match Problem

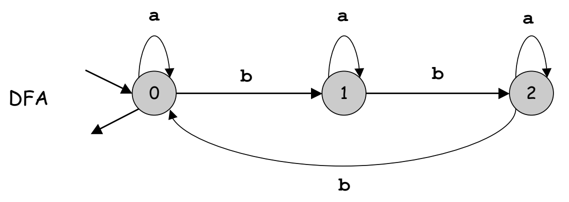
Regular expressions are a concise way to describe patterns.
- How would you implement `String.matches` ?
- Hardware: build a deterministic finite state automaton (DFA).
- Software: simulate a DFA.

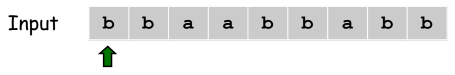
DFA: simple machine that solves the pattern match problem.
- Different machine for each pattern.
- Accepts or rejects string specified on input tape.
- Focus on `true` or `false` questions for simplicity.



Deterministic Finite State Automaton (DFA)

Simple machine with N states.
- Begin in start state.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- Accept input string if unlabeled arc leaves last state.

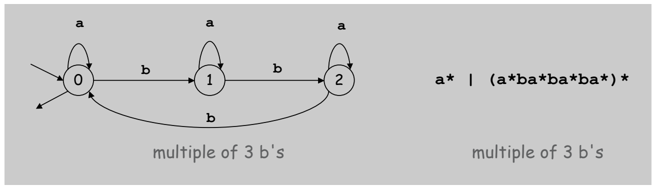




Theory of DFAs and REs

RE. Concise way to describe a set of strings.
DFA. Machine to recognize whether a given string is in a given set.

Duality: for any DFA, there exists a regular expression to describe the same set of strings; for any regular expression, there exists a DFA that recognizes the same set.



Practical consequence of duality proof: to match regular expression patterns, (i) build DFA and (ii) simulate DFA on input string.

Implementing a Pattern Matcher

Problem. Given a RE, create program that tests whether given input is in set of strings described.

Step 1: build the DFA.

- A compiler!
- See COS 226 or COS 320.

Step 2: simulate it with given input. Easy.

```
State state = start;
while (!StdIn.isEmpty()) {
   char c = StdIn.readChar();
   state = state.next(c);
}
System.out.println(state.accept());
```

Application: Harvester

Harvest information from input stream.

- Harvest patterns from DNA.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg
```

- Harvest email addresses from web for spam campaign.

```
% java Harvester "[a-z]+@([a-z]+\\.)+(edu|com)"
      http://www.princeton.edu/~cos126
rs@cs.princeton.edu
dgabai@cs.princeton.edu
wayne@cs.princeton.edu
```

Application: Harvester

Harvest information from input stream.

- Use `Pattern` data type to compile regular expression to NFA.
- Use `Matcher` data type to simulate NFA.

equivalent, but more efficient representation of a DFA

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester {
   public static void main(String[] args) {
      String re         = args[0];
      In in             = new In(args[1]);
      String input      = in.readAll();
      Pattern pattern = Pattern.compile(re);
      Matcher matcher = pattern.matcher(input);
      while (matcher.find()) {
         System.out.println(matcher.group());
      }
   }
}
```

Application: Parsing a Data File

Ex: parsing an NCBI genome data file.

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus (platypus)
ORIGIN
     1 tgtatttcat ttgaccgtgc tgttttttcc cggtttttca gtacggtgtt agggagccac
    61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
   121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
   ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

```
String re = "[ ]*[0-9]+([actg ]*).*";
Pattern pattern = Pattern.compile(re);
In in = new In(filename);
while (!in.isEmpty()) {
   String line = in.readLine();
   Matcher matcher = pattern.matcher(line);
   if (matcher.find()) {
      String s = matcher.group(1).replaceAll(" ", "");
      // do something with s
   }
}
```

extract the RE part in parentheses

replace this RE

with this string

Fundamental Questions

Q. Which languages cannot be described by any RE?
A.

- Bit strings with equal number of 0s and 1s.
- Decimal strings that represent prime numbers.
- Genomic strings that are Watson-Crick complemented palindromes.
- Many more....

Q. How can we extend REs to describe richer sets of strings?
A. Context free grammar (e.g., Java).

Reference: http://java.sun.com/docs/books/jls/second_edition/html/syntax.doc.html

Q. How can we make simple machines more powerful?

Q. Are there any limits on what kinds of problems machines can solve?

Summary

Programmer.
- Regular expressions are a powerful pattern matching tool.
- Implement regular expressions with finite state machines.

Theoretician.
- Regular expression is a compact description of a set of strings.
- DFA is an abstract machine that solves pattern match problem for regular expressions.
- DFAs and regular expressions have limitations.

You. Practical application of core CS principles.

Variations.
- Terminology: DFA, FSA, FSM.
- DFAs with output: Moore machines, Mealy machines.

7.5: Turing Machines

Challenge. Design simplest machine that is "as powerful" as conventional computers.

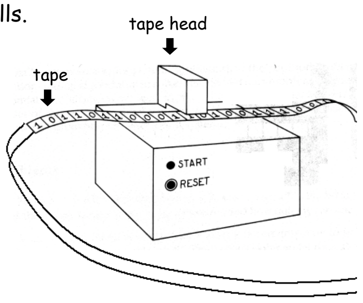
Alan Turing (1912-1954)

Turing Machine: Components

Alan Turing sought the most primitive model of a computing device.

Tape.
- Stores input, output, and intermediate results.
- One arbitrarily long strip, divided into cells.
- Finite alphabet of symbols.

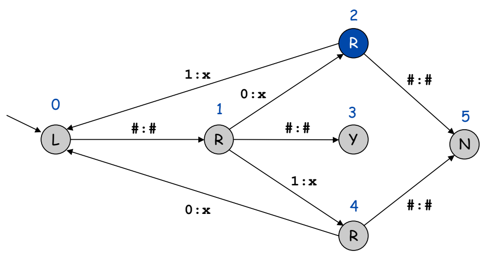
Tape head.
- Points to one cell of tape.
- Reads a symbol from active cell.
- Writes a symbol to active cell.
- Moves left or right one cell at a time.



Turing Machine: Fetch, Execute

States.
- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

State transition diagram.
- Ex. if in state 2 and input symbol is `1` then: overwrite the `1` with `x`, move to state 0, move tape head to left.



Before

…	#	#	x	x	x	1	1	0	#	#	…

Turing Machine: Fetch, Execute

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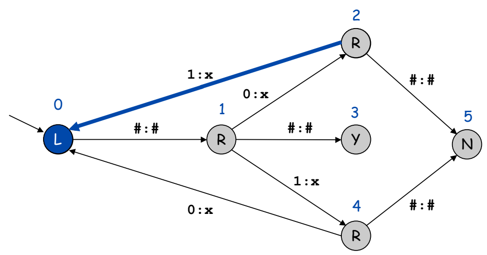


After

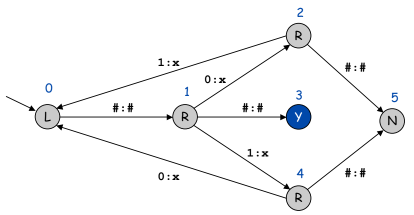
…	#	#	x	x	x	x	1	0	#	#	…

Turing Machine: Initialization and Termination

Initialization.
- Set input on some portion of tape.
- Set tape head.
- Set initial state.

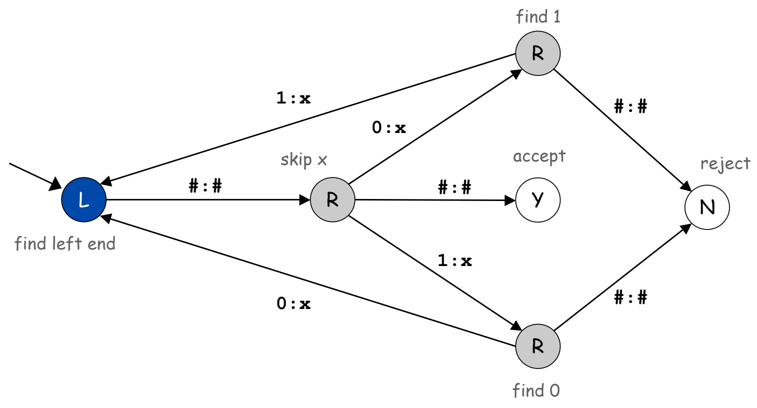
…	#	#	0	0	1	1	1	0	#	#	…

Termination.
- Stop if enter `yes`, `no`, or `halt` state.
- Infinite loop possible.



…	#	#	x	x	x	x	x	x	#	#	…

Example: Equal Number of 0s and 1s



…	#	#	0	0	1	1	1	0	#	#	…

Turing Machine Summary

Goal: simplest machine that is "as powerful" as conventional computers.

Surprising fact 1. Such machines are very simple.
Surprising fact 2. Some problems cannot be solved by ANY computer.

next lecture

Consequences.

- Precursor to general purpose programmable machines.
- Exposes fundamental limitations of all computers.
- Enables us to study the physics and universality of computation.
- No need to seek more powerful machines!

Variations.

- Halt state: accept string if computation halts; reject otherwise.
- TM with output (the tape).

Alan Turing

Alan Turing (1912-1954).

- Father of computer science.
- Computer Science's "Nobel Prize" is called the Turing Award.

It was not only a matter of abstract mathematics, not only a play of symbols, for it involved thinking about what people did in the physical world.... It was a play of imagination like that of Einstein or von Neumann, doubting the axioms rather than measuring effects.... What he had done was to combine such a naïve mechanistic picture of the mind with the precise logic of pure mathematics. His machines – soon to be called Turing machines – offered a bridge, a connection between abstract symbols, and the physical world. -John Hodges

Alan Turing (left)
Elder brother (right)