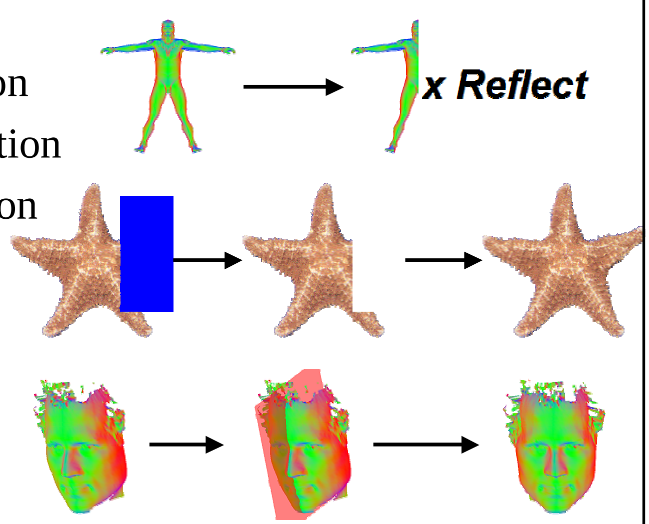
Symmetry Detection

Motivation

- Compression
- Reconstruction
- Classification
- Analysis
- Alignment
- Matching
- Etc.

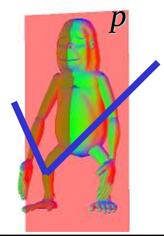


Definition

A collection of:

points/lines/curves/triangles/surfaces/volumes,

has *reflective symmetry* w.r.t. some plane *p* if the reflection Ref_p through *p* fixes the collection.

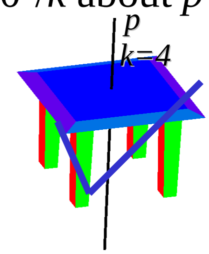


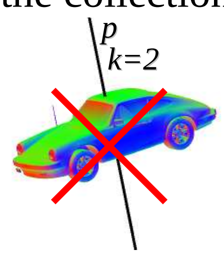
Definition

A collection of:

points/lines/curves/triangles/surfaces/volumes,

has *rotational symmetry* of order *k* w.r.t. some axis *p* if the rotation Rot_p^k by an angle of 360°/*k* about *p* fixes the collection.



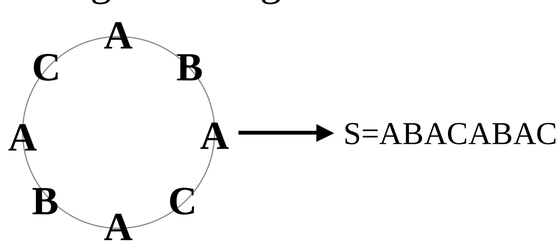


Outline

- Geometry based approaches
- Geometry/descriptor based approaches
- Descriptor based approaches

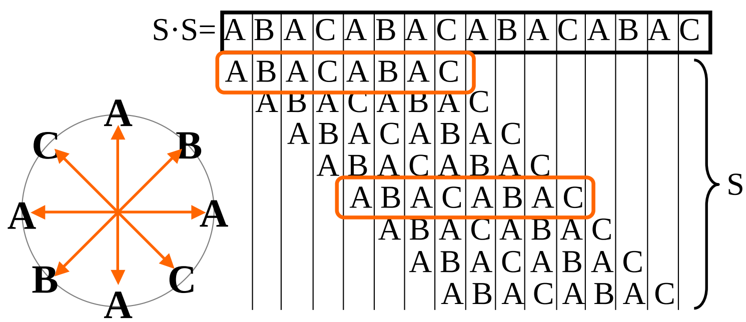
Discrete Symmetry

String matching



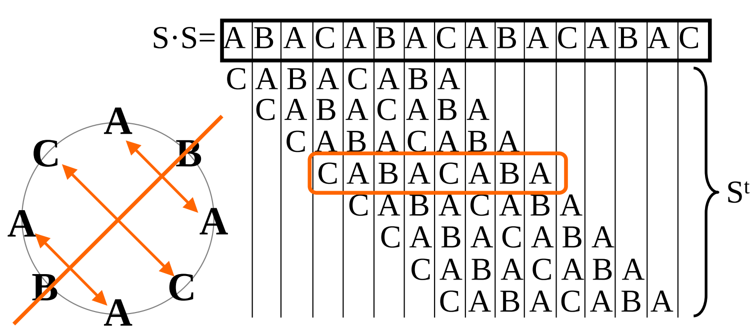
Discrete (Rotational) Symmetry

Search for non-trivial repeating patterns in concatanation (S_ S·S):



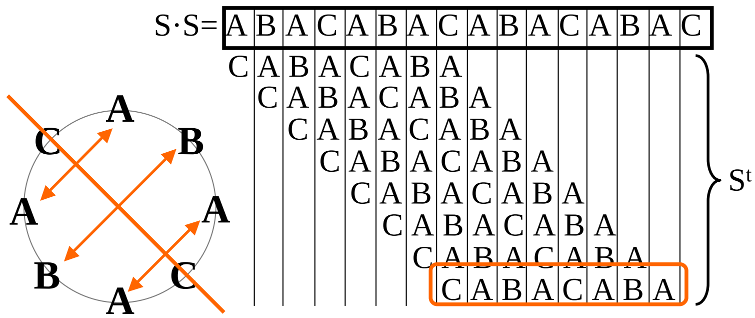
Discrete (Reflective) Symmetry

Search for non-trivial repeating patterns in concatanation (S^t_ S·S):



Discrete (Reflective) Symmetry

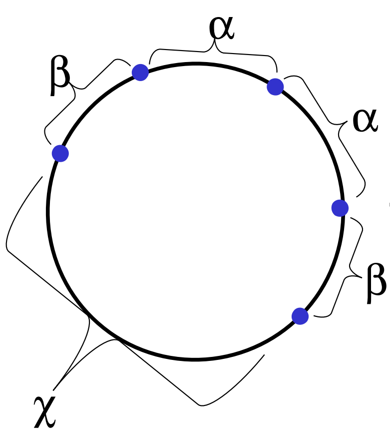
Search for non-trivial repeating patterns in concatanation (S^t_ S·S):



Outline

- Geometry based approaches
- Geometry/descriptor based approaches
- Descriptor based approaches

Points on a Circle



Sort by angle and compute angle between adjacent points

→ S=ααβχβ

Generate string from ordered list of angles

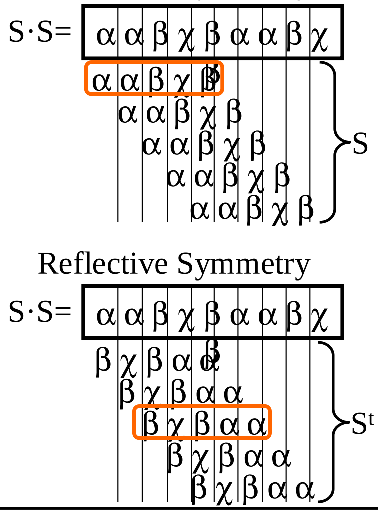
IEEE, 1985. Atallah

The Visual Computer, 1985. Wolter et al.

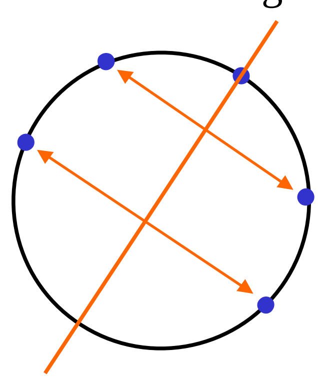
Information Processing Letters, 1986. Highnam

Points on a Circle

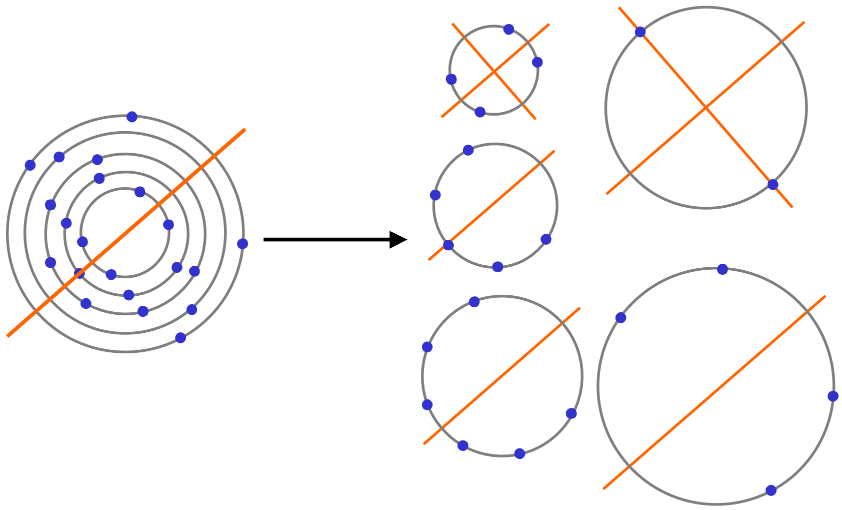
Rotational Symmetry



Find rotational and reflective symmetries of string:



Points in 2D



Discussion

Pro:

- Evaluates all symmetries

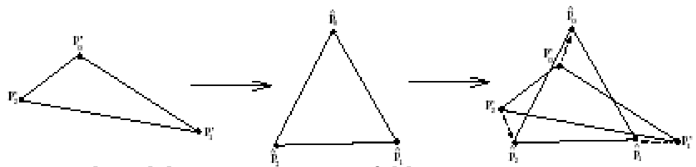
Cons:

- Only perfect symmetry
- Does not generalize to 3D

Continuous Measure of Symmetry	No
Identifies All Symmetries	**Yes**
3D	No

Symmetry Distance

Measure of symmetry as distance to nearest symmetric model:

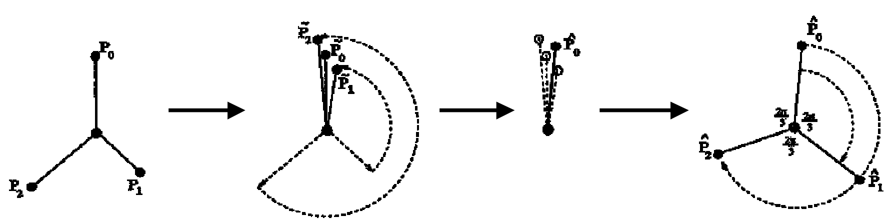


Initial model | Nearest 3-fold symmetric | Symmetry Distance

IWVF, 1994. Zabrodsky et al.
IEEE, 1995. Zabrodsky et al.

Symmetry Distance

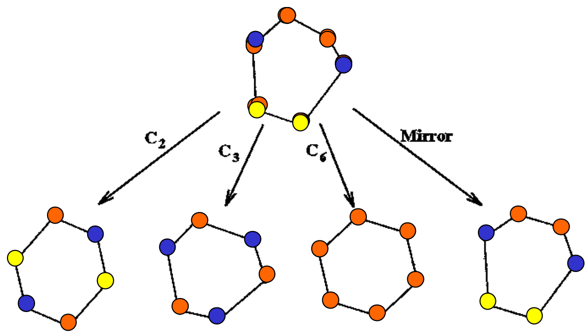
Nearest symmetric model can be obtained by folding:



IWVF, 1994. Zabrodsky et al.
IEEE, 1995. Zabrodsky et al.

Symmetry Distance

Needs establishment of correspondences



IWVF, 1994. Zabrodsky et al.
IEEE, 1995. Zabrodsky et al.

Discussion

Pro:

- A continuous measure of symmetry
- Generalizes to 3D

Cons:

- Does not identify potential symmetries
- Depends on establishing of correspondences

Continuous Measure of Symmetry	**Yes**
Identifies All Symmetries	No
3D	**Yes**

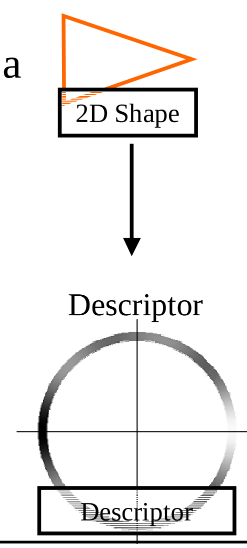
Outline

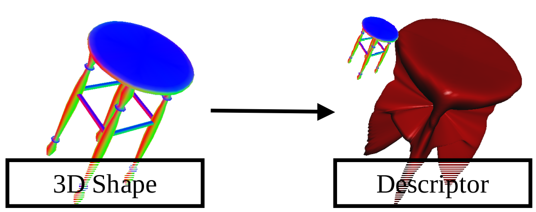
- Geometry based approaches
- Geometry/descriptor based approaches
- Descriptor based approaches

Approach

Leverage shape descriptor to obtain a structured shape representation:

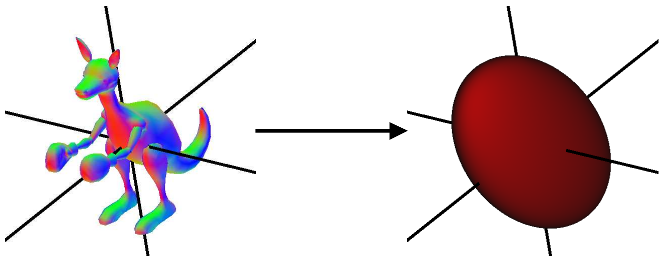
- Correspondences become implicit





PCA

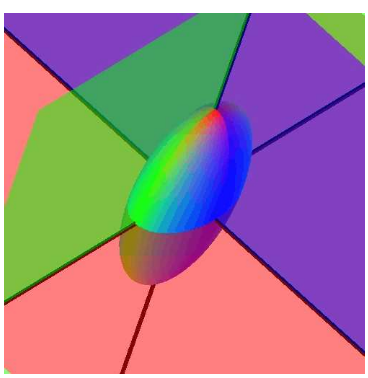
PCA of a 3D model gives an ellipsoid:



IEEE TENCON, 1996. O'Mara et al.
IEEE, 1997. Sun et al.

Symmetry of the Ellipsoid

Planes of reflective symmetry of an ellipsoid are perpendicular to principal axes



IEEE TENCON, 1996. O'Mara et al.
IEEE, 1997. Sun et al.

Symmetry of the Ellipsoid

Axes of rotational symmetry of an ellipsoid:

- Are principal axes
- Only occur if the other two axes have the same length (when orderg 2)

IEEE TENCON, 1996. O'Mara et al.
IEEE, 1997. Sun et al.

Computing Symmetry with PCA

1. Compute the principal axes of a model
2. Identify candidate axes/planes of symmetry
3. Evaluate quality of candidates by comparing the shape descriptor (SD) of initial models with the SDs of the rotations/reflections.

IEEE TENCON, 1996. O'Mara et al.
IEEE, 1997. Sun et al.

Discussion

Pros:

- A continuous measure of symmetry
- Generalizes to 3D
- Addresses the correspondence issue

Con:

- Evaluates each symmetry type independently

Continuous Measure of Symmetry	**Yes**
Evaluates All Symmetries	No
3D	**Yes**

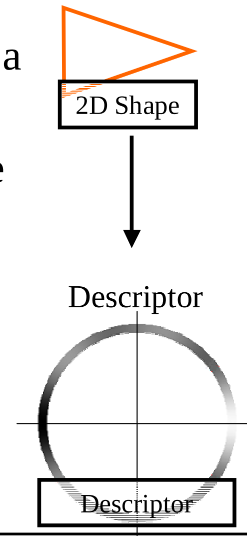
Outline

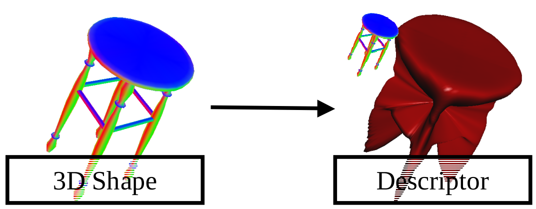
- Geometry based approaches
- Geometry/descriptor based approaches
- Descriptor based approaches

Approach

Leverage shape descriptor to obtain a structured shape representation:

- Possibility for efficient exhaustive symmetry detection

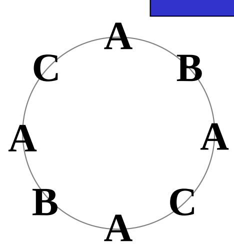


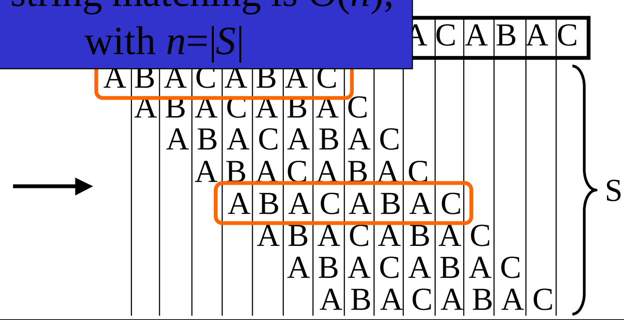


Observation

Sub-string based symmetry detection is efficient but is binary.

Sub-string matching is O(n), with n=|S|



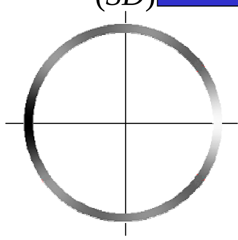


Circular Function Descriptors

Replace discrete matching of circular string with correlation of circular function

Correlation is O(b log(b)), with $|SD|$=O(b)

Shape Desc
(SD)



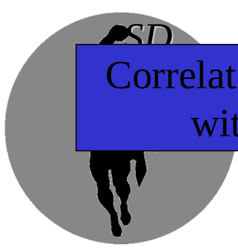
$$\longrightarrow Sym(SD,\alpha)=\int_0^{2\pi} SD(t-\alpha)SD(t)\,dt$$

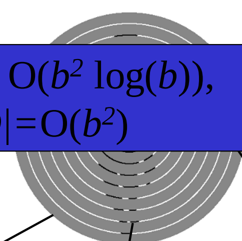
PRL, 1995. Sun

RTI, 1999. Sun et al.

2D Function Descriptors

Generalize to 2D functions by looking at circular restrictions





Correlation is O(b^2 log(b)), with $|SD|$=O(b^2)

$$Sym(SD,\gamma)=\overbrace{r_1}^{\text{scale factor}} Sym(\cdot,\gamma)+\overbrace{r_2}^{\text{scale factor}} Sym(\cdot,\gamma)+\overbrace{r_3}^{\text{scale factor}} Sym(\cdot,\gamma)+\ldots$$

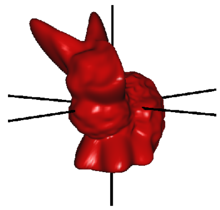
ECCV, 2002. Kazhdan et al.

Spherical Function Descriptors

Use spherical harmonics and Wigner-D transform to perform rotational correlation.

Shape Descriptor (SD)

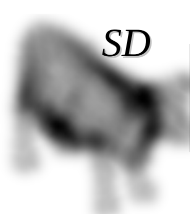
Correlation is $O(b^4)$, with $|SD|=O(b^2)$



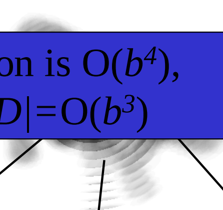
$$Sym(SD,R) = \int_{Sphere} SD(R(t))R(t)dt$$

3D Function Descriptors

Generalize to 3D functions by looking at spherical restrictions



Correlation is $O(b^4)$, with $|SD|=O(b^3)$



$$Sym(SD,R) = \overbrace{r_1}^{\text{scale factor}} Sym(\ ,R) + \overbrace{r_2}^{\text{scale factor}} Sym(\ ,R) + \overbrace{r_3}^{\text{scale factor}} Sym(\ ,R) + \ldots$$

ECCV, 2002. Kazhdan et al.

Summary

- Use shape descriptors for efficiency and simplicity
- Generalize fast sub-string matching to FFT
- Extend FFT to FST

	Sub-String	Symmetry Distance	PCA	FFT	FST
Continuous Measure of Symmetry	No	**Yes**	**Yes**	**Yes**	**Yes**
Identifies all Symmetries	**Yes**	No	No	**Yes**	**Yes**
3D	No	Yes	**Yes**	No	**Yes**