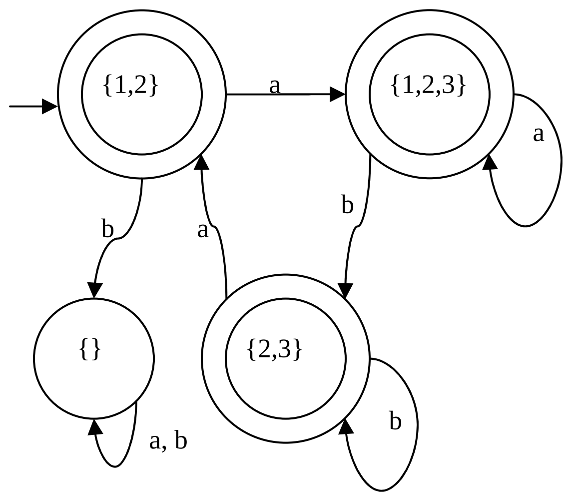
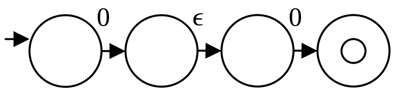
COS 487
Fall 2003

Problem Set 2 Solutions

Problem 1:



Problem 2:



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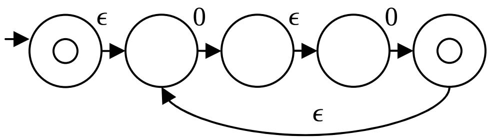
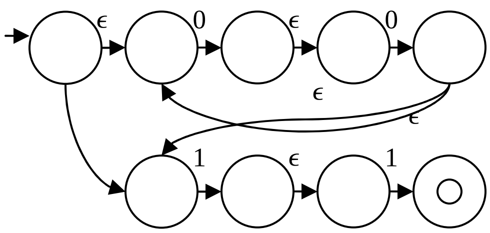
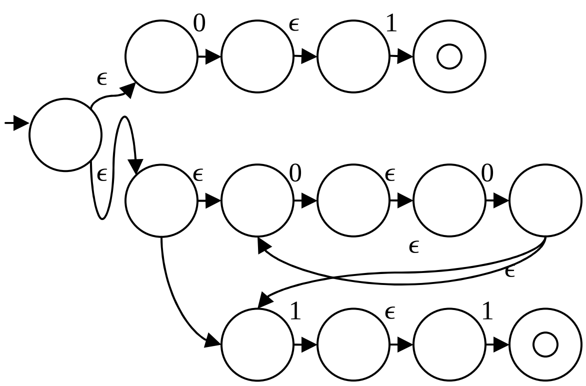


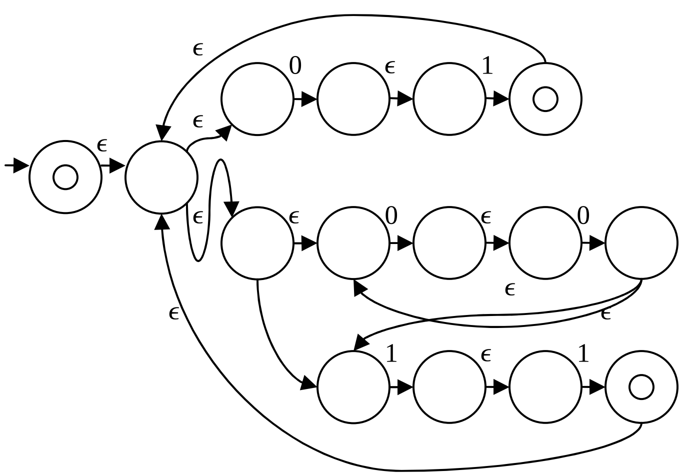
(00)*



(00)*11



(00)*11∪ 01



((00)*11∪ 01)*

Problem 3:

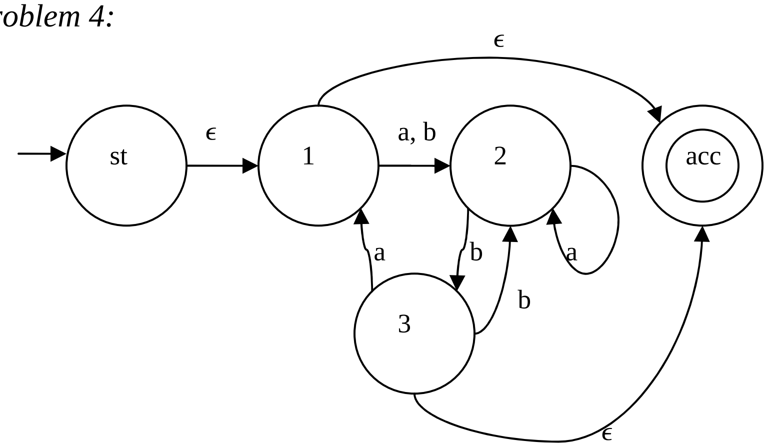
(b:) Members: ab, abab. Non-members: ϵ, a.

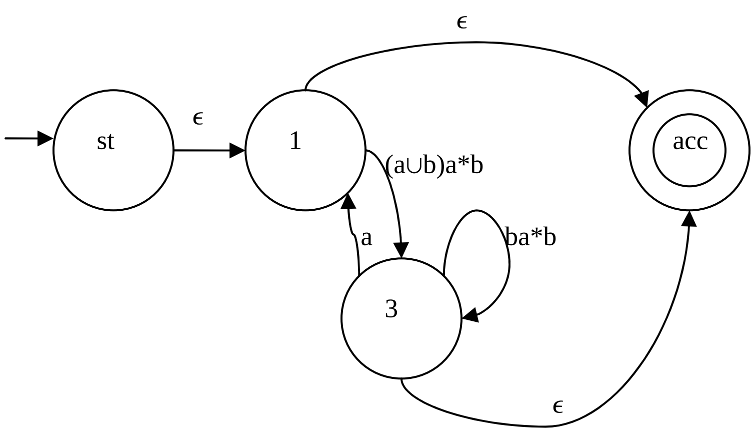
(d:) Members: ϵ, aaa. Non-members: a, b.

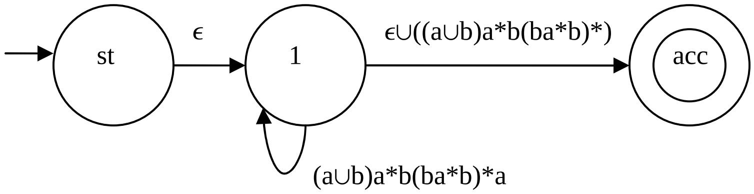
(f:) Members: aba, bab. Non-members: a, b.

(h:) Members: a, ab. Non-members: ϵ, b.

Problem 4:







ϵ∪((a∪b)a*b(ba*b)*)∪((a∪b)a*b(ba*b)*a)*((a∪b)a*b(ba*b)*)

Problem 5:

Let A_k be the language of all strings ending in k-1 zeroes. Each A_k is recognized by a DFA with k states – a state counts how many tailing zeroes have been seen (0 to k-1). Suppose A_k is recognized by a DFA with k-1 states. Then, on the input 0^{k-1}, some state repeats twice, so we may eliminate the part of the string that takes the DFA from this state to itself. Then, the DFA must accept some string of length less than k-1, a contradiction.

Problem 6:

Let D = (Q, Σ, δ, q_0, F) be a DFA which recognizes A. We give an NFA N for $A_{1/2-}$. The states of N will be (Q x Q x Q), plus a start state. From the start state we place an epsilon arrow to every state (q_0, p, p), for p in Q and guess that D will be in state p after reading the input. From every other state, we use the following transition function:

δ'((q, r, p), s) = {δ(q, s), r', p | δ(r,t) = r' for some t in Σ.}

We accept a state (q, r, p) if q and p are equal (D did end in state p) and if r is an accept state (some string of the same length as the input takes D from the state p to an accept state).

Problem 7:

Suppose D is a DFA which accepts L. Let p be a prime number larger than the pumping length of D. Then, 1^p may be divided into three pieces, $1^p = xyz$, $|y| > 0$, and xy^iz is in L for all non-negative i. Then, $xy^{1+p}z$ is in L, but $|xy^{1+p}z|$ is strictly larger than p and divisible by p, so its length is composite, a contradiction.