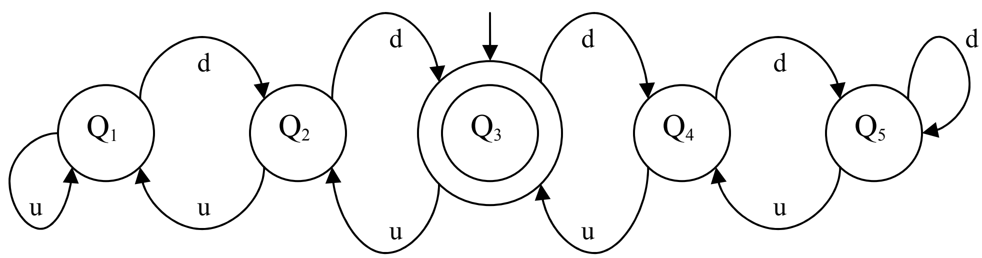
COS 487
Fall 2003

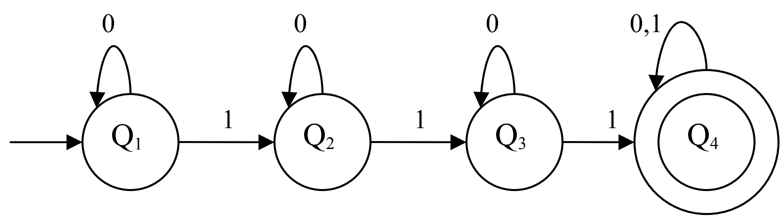
Problem Set 1 Solutions

Problem 1:

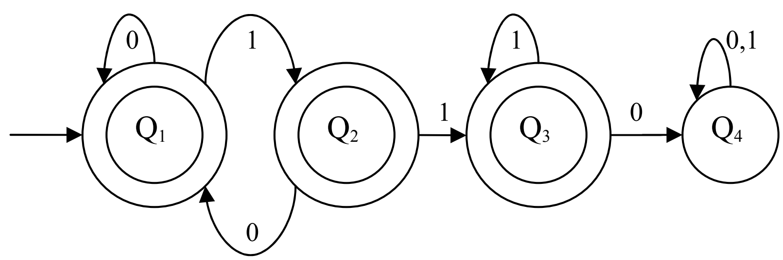


Problem 2:

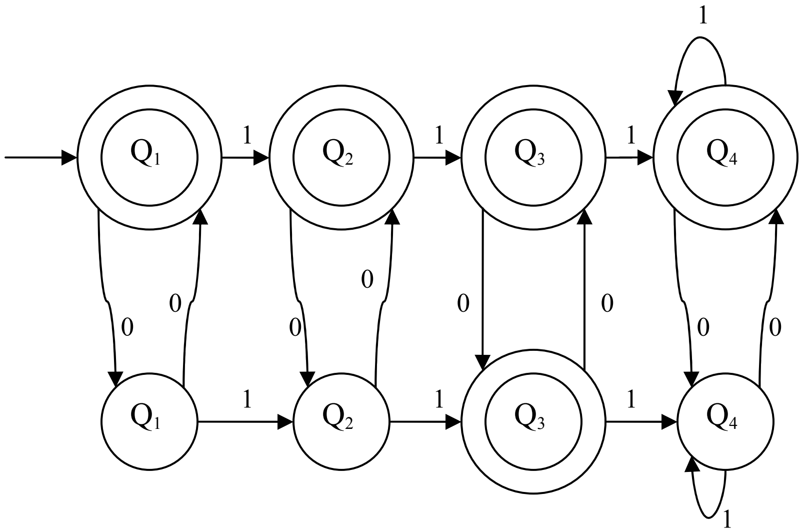
(b:)



(f:)



(l:)



Problem 3:

(b:) $0^*10^*10^*1\Sigma^*$

(f:) $(0 \cup (10))^*1^*$

(l:) $1^* \cup (1^*01^*01^*)^* \cup (0^*10^*10^*)$

Problem 4:

For all parts, we assume that A_n is regular with pumping length p, and derive a contradiction.

(a:) Using the pumping lemma on the input $0^p1^p2^p$ shows that $0^{p+n}1^p2^p$ must also be in A_1 for some n>0.

(b:) Using the pumping lemma on the input $a^pba^pba^pb$ shows that $a^{p+n}ba^pba^pb$ must also be in A_2 for some n>0.

(c:) Using the pumping lemma on the input $a^{2\wedge p}$ shows that $a^{(2\wedge p)+n}$ must also be in A_3 for some p>n>0. But, $2^p < 2^p + n < 2^{p+1}$, so this string cannot be in A_3.

Problem 5:

If *ADD* is regular with pumping length p, we may apply the pumping lemma to $1^p=1^p+0$, showing that $1^{p+n}=1^p+0$ is in *ADD* for some n>0.

Problem 6:

Observe that any binary string beginning and ending with the same digit has an equal number of occurrences of the substrings 01 and 10. Thus, the following regular expression recognizes *D*: $\varepsilon \cup 0\Sigma^{*}0 \cup 1\Sigma^{*}1$.

Problem 7:

Let *A* be the language given by the regular expression $(01)^{*}0$. Then, $A_{1/3-1/3} = \{(01)^{n}00(10)^{n} \mid n \geq 0\}$. If p is the pumping length of $A_{1/3-1/3}$, applying the pumping lemma to $(01)^{p}00(10)^{p}$ yields a string which has a pair of consecutive zeroes past the center of the string, a contradiction.