Problem Set 1 Solutions

Problem 1:

Problem 2:

(b:)

(f:)

0 0 0,1

Q1 Q2 Q3 Q4

0 1 0 0,1

Q1 Q2 Q3 Q4

0 1 0 0,1

Q1 Q2 Q3 Q4
Problem 3:

(b): \(0^*10^*10^*1\Sigma^*

(f): \((0 \cup (10))^*1^*

(l): \(1^* \cup (1^*01^*01^*)^* \cup (0^*10^*10^*)

Problem 4:

For all parts, we assume that \(A_n\) is regular with pumping length \(p\), and derive a contradiction.

(a:) Using the pumping lemma on the input \(0^p1^p2^p\) shows that \(0^p+1^p2^p\) must also be in \(A_1\) for some \(n>0\).

(b:) Using the pumping lemma on the input \(a^pba^pba^pb\) shows that \(a^p+na^pba^pba^pb\) must also be in \(A_2\) for some \(n>0\).

(c:) Using the pumping lemma on the input \(a^{2^p}\) shows that \(a^{(2^p)+n}\) must also be in \(A_3\) for some \(p>n>0\). But, \(2^p < 2^p + n < 2^{p+1}\), so this string cannot be in \(A_3\).

Problem 5:

If \(ADD\) is regular with pumping length \(p\), we may apply the pumping lemma to \(1^p=1^p+0\), showing that \(1^p+n=1^p+0\) is in \(ADD\) for some \(n>0\).
Problem 6:

Observe that any binary string beginning and ending with the same digit has an equal number of occurrences of the substrings 01 and 10. Thus, the following regular expression recognizes $D$: $\varepsilon \cup 0\Sigma^*0 \cup 1\Sigma^*1$.

Problem 7:

Let $A$ be the language given by the regular expression $(01)^*0$. Then, $A_{1/3-1/3} = \{(01)^n00(10)^n \mid n \geq 0\}$. If $p$ is the pumping length of $A_{1/3-1/3}$, applying the pumping lemma to $(01)^p00(10)^p$ yields a string which has a pair of consecutive zeroes past the center of the string, a contradiction.