Life and Work of Wilhelm Cauer (1900 – 1945)

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Abstract

The paper concerns Wilhelm Cauer (1900-1945), a German mathematician and scientist who left an opus ranging from mathematics and mathematical physics to electrical engineering and network synthesis. It focuses on the latter part of Cauer's scientific work and the problems he experienced in attempting to establish a new interdisciplinary field of research and work as a scientist at a time marked by the rise of Nazism, the forced exodus of large sections of the scientific community, and later by the end of World War II.

1 Introduction

Many contributions to the analysis of electrical networks have been made by Helmholtz, Maxwell, Heaviside, Steinmetz, Kennelly and others on the basis of the discoveries of Ohm (1827) and Kirchhoff (1845-1847), and it was in this way that network theory emancipated from analytical mechanics and electrophysics before 1900 as the first basic branch of electrical engineering. At the beginning of this century, a few engineers began studying the design aspects of communication systems. The most prominent researchers, G. A. Campbell in the United States and K. W. Wagner in Germany, explored selective filter circuits for telephone applications in the early years of World War I, thus paving the way for the first ideas about electrical circuit synthesis. They designed sophisticated filter circuits simply by cascading elementary sections of lossless inductors and capacitors. The circuits obtained in this way had a striking similarity to circuit models in engineering transmission line theory and henceforth were called 'wave filters'. For a historical account of early network theory, see [2], [27], [36].

Around 1920 electrical engineers were able to analyse the behavior of certain filter networks and proved some theorems about the properties of attenuation curves. However, there were no results on the question what filter characteristics are realizable and, if so, how to find a physical realization. The first breakthrough towards a systematic synthesis of filters was achieved in 1924 by R. M. Foster in his celebrated paper *A reactance theorem* [31]. W. Cauer immediately recognized the potentialities of Foster's result. In his doctorate thesis (1926) on *The realization of impedances of specified frequency dependence* [8] he presented a precise mathematical analysis of the problem and performed the first steps towards a scientific program that condensated the rather "wild" engineering design tasks into three clear-cut classes of problems concerning the

- realizability
- approximation
- realization and equivalence

of network or transfer functions. This contribution to the systematic design of electrical filters was tantamount to the very beginning of network synthesis. Cauer's program is explicitly formulated in the lecture he held in 1928 on the occasion of his habilitation in Göttingen; an extended version was published in 1930 [12].

When Cauer formulated his *program*, it was an audacious undertaking. During his time engineering problems were typically solved in an empirical fashion by analysing specific circuits instead of looking at *classes* or *families* of circuits. In this sense Cauer was also a protagonist of mathematical system theory *ante litteram*, and it seems not too hazardous to compare the importance of Cauer's program for network synthesis to that of Felix Klein's famous 'Erlanger Programm' for modern geometry and mathematical physics.

2 Wilhelm Cauer: Son of a Distinguished Berlin Family (by Emil Cauer)

Wolfgang Mathis and Rainer Pauli pay tribute to my father's contribution to the development of networks and systems theory. It seems likely that after World War II this contribution was lost sight of not only because Cauer was neither enough of a mathematician nor enough of a physicist to be regarded as an outstanding scientist in either of the two disciplines, but also because his life and work in general were somewhat fragmented. I think that knowledge of his life will lead to an improved assessment of his achievement, and this is why I would like to summarise his biography – a hundred years after his birth.

Wilhelm Cauer was born in Berlin on June 24, 1900, the sixth child of the family. His mother came from a family of preachers and teachers. His father, also called Wilhelm, was a Privy Councilor and professor of railway engineering at the Technical University of Berlin.

Of four generations of male ancestors nearly all had been academically trained. At the turn of the century, the Cauers were a family closely linked to many well-known scholars. The first grammar school my father attended had actually been founded by his great-grandfather. The street in which it was located was called Cauerstrasse. This gives some idea of the encouragement as well as the demands that Wilhelm Cauer experienced in his youth. In general, the family had a high and idealistic belief in the value of education. Even my grandfather's sisters and their step-mother published books while and were active in the women's liberation movement. In a time when some professors still did not tolerate female students, three of my father's sisters completed their doctor's thesis. But at the same time, the males were distanced from the social problems of those times. They were patriotic in an uncomplicated fashion and believed that science and learning were completely independent of politics. They had a high opinion of individual freedom as well as of individual duty to the 'Fatherland'.

When Wilhelm Cauer was thirteen, he became determined to study mathematics. He used to meditate for hours on mathematical and chess problems. After a few months of service in the army at the end of World War I he started to study at the University of Berlin. In 1921 he continued in Bonn where H. Beck, one of his former teachers of the Mommsen Gymnasium, Berlin, had become professor in mathematics. I should like to agree with some lines which H. Beck wrote in 1920: "My dear Mr Cauer: That's a bit too much – to expect you to become a school teacher. You would be eaten alive by the boys. [...] I tell you what you should become – a professor, for this profession has the least to do with life. [...] I told your father six or seven years ago that you were a born professor."

At that time even his scholarly father mentioned in one of his letters the risk of social isolation of his son. At the same time, his fiancé, Karoline, encouraged him to take life less seriously. The purpose and goal of his concentration on studies as well as the situation in Germany of that time is made clear by a simple sentence Cauer wrote to his mother from Bonn: "And by the way, the prospects for getting a chair are not at all bad – for non-Jews, that is." Twelve years later, in 1933, the goal of the rising generation had taken on a very different context, and this unsuspecting observation of anti-Semitism soon came to be confirmed by a reign of terror which he had not foreseen.

In 1922 he met Max von Laue and began to work in the area of general relativity. I do not know why von Laue chose to discontinue a working relationship. Cauer's first publication was a contribution to the general theory of relativity, and was published in 1923. Then he started to study problems in electrical engineering at the Technical University of Berlin. In 1924 he graduated in applied physics and entered the employ of Mix & Genest, a Berlin company working in the area of communication and telephone systems, then a branch of Bell Telephone Company. There he worked on probability theory as applied to telephone switching systems and calculations relating to time-lag relays.

After completing two publications, one on telephone switching systems and the other on losses of real inductors, Cauer began to study the problem of filter design. Due to his interest in this field, he regularly corresponded with Foster, who was also working on the same problem.

While working as a research assistant, he presented his thesis paper in June 1926 to G. Hamel, head of the Institute of Applied Mathematics and Mechanics at the Technical University of Berlin. The second referee was K.W. Wagner.

In 1927 he contacted Richard Courant in Göttingen as well as Vannevar Bush of MIT because he was interested in the construction of computing machines capable of solving systems of linear equations. Thus, he became a research assistant at Courant's Institute of Mathematics at the University of Göttingen. Subsequently, he got his habilitation (academic teaching license) and became an external university lecturer in 1928. Due to the economic crisis however, his family could not solely live from this position. Now and then some extra money from royalties meant help to the young couple who had married in 1925 and now had a child.

In 1930 the Rockefeller Foundation granted Cauer a one year scholarship for studies at MIT and Harvard University, and his wife followed him to the United States. There he became acquainted with several American scholars working in network theory and mathematics. He was a member of the team around Vannevar Bush, who was the developer of several electrical and mechanical machines for the solution of mathematical problems. It was also there that he completed the *Siebschaltungen* [14]. After his term at MIT, Cauer worked for three months for the Wired Radio Company in Newark, N.J. During the turbulent years which followed, my parents often recalled this impressive year in America, which broadened their outlook and made them critical of German visions of omnipotence. Their American friendships outlasted the Second World War and brought help to our family after the defeat of Nazism.

Back in Göttingen, Germany, the sheer lack of funding caused by the Depression prevented Cauer from finishing the development of an electrical calculating machine that would have been the fastest linear systems solver at the time (20 minutes for 10 unknowns with an accuracy of maximally 4 digits; for a photograph see [7]). Then, in early 1933, the demented apparatus of the Third Reich took control of Göttingen University. A racist, right-wing revolution swept through the entire university. Admittedly, the university received orders from the Hitler government in Berlin, but there were a large number of people who were only too ready to obey these orders, although they were not really forced to do so. The small town of Göttingen, which owed its reputation to the many famous scholars that had taught at its university, became 'cleansed of the Jews'. Nearly 70% of the teaching staff of the world-famous Mathematics Institute there were either Jewish or of Jewish descent. The elite, including its director, Richard Courant, had to leave. Young Nazi leaders organized student riots and summoned the remaining staff to participate in 'voluntary' military sports camps. Cauer thought that he could come to terms with the regime by spending periods of time in sports camps of this kind. But in those days of hysterical investigations it soon became known that one of Cauer's ancestors had been a certain Daniel Itzig (1723-1799), a banker of the prussian king Frederick II, among whose descendents were a number of well-known bankers, statesmen and composers. Although this did not mean that my father was going to be affected by the Nazi race laws, he was given to understand that there was no future for him at the University of Göttingen.

A further problem at that time was that few people could appreciate the vast potential of Cauer's special field of work. Whenever faculties discussed making new professional appointments, they tended to look for teachers in the traditional fields of mathematics. They did not pay attention to Cauer because, for mathematicians, he seemed too involved in applied sciences, and for electrical engineers his contributions included too much mathematics. Therefore, although Cauer was nominally granted the title of professor in 1935, no chair was actually available for him. It took him quite a long time to realize that his life goal of an academic career would not work out. He had a family of three children and their mother to keep, and his small income was quite inadequate. He even made attempts to obtain a teaching position in the United States, but to no avail. In 1936, after various attempts to gain a position in the industry, and a short term appointment in Kassel with the aircraft manufacturer Fieseler & Storch, be became director of the laboratory at Mix & Genest. This situation in Berlin gave him stimulation and scope, and also enabled him to give lectures on applied mathematics at the Technical University in Berlin beginning in 1939.

In 1941, the first volume of his main work, *Theorie der linearen Wechselstromschaltungen* [22] was printed, in which he provided detailed information on what he intended to publish in the second volume. The manuscript to this was destroyed by Allied bombing in 1943, however, and he started anew. He actually completed this second manuscript, but it was either destroyed or taken by the Red Army from the safes at Mix & Genest in 1945.

Born in 1932 as one of six children, I remember my mother ruling over the family, while my father, home from his office, would withdraw into his study to work on mathematical formulae. He even used to work while we crouched in our airraid shelter during World War II bombing attacks. I would not say he was a workaholic, but that he was dedicated to his work and inclined to be single-minded. He did not like to waste time, and he distinguished the important from the unimportant. This helped him to be broadminded and generous.

He was warm and gentle towards me as a child, though sparing of words. He liked to play chess with me, patiently waiting for me to make my helpless moves and briefly explaining where I had gone wrong. I remember with particular affection the evening hours when the family would sit in his study whilst he read out loud in German from foreign books such as 'Robinson Crusoe' by Daniel Defoe.

I never heard him repeat the Nazi slogans that we were compelled to hear at school and all around us in daily life. I am sure he was aware that the war against the Soviet Union and the United States as well as the exploitation of most European countries would lead into catastrophe. On the other hand, he was hesitant to make any criticism of the regime, at least in front of us children, in order to protect us from Nazi investigations. We knew of Theresienstadt, but not of Auschwitz. My parents used to invite the research students who were working on their doctorate to our house, but I remember my father avoiding small talk or airing his private opinions when we had guests. I am sure they found him a modest and gentle host.

In January and March 1945, when it became clear which of the Allied Powers would capture the various parts of Germany, my father deposited some of his papers with friends in Göttingen. The last time I saw my father was two days before the American Forces occupied the small town of Witzenhausen in Hesse, about 30 km from Göttingen. We children were staying there with relatives in order to protect us from air raids. Because rail travel was already impossible, my father was using a bicycle. Military Police was patrolling the streets stopping people and checking their documents. By that time, all men over 16 were forbidden to leave towns without a permit, and on the mere suspicion of being deserters, many were hung summarily in the market places. Given this atmosphere of terror and the terrible outrages which Germans had inflicted on the peoples of the Soviet Union, I passionately tried to persuade my father to hide rather than return to Berlin, since it was understandable that the Red Army would take its revenge. But he decided to go back, perhaps out of solidarity with his colleagues still in Berlin, or just due to his sense of duty, or out of sheer determination to carry out what he had decided to do.

Seven months after the ending of that war, my mother succeeded in reaching Berlin and found the ruins of our house in a southern suburb of the city. None of the neighbors knew about my father's fate. But someone gave identification papers to my mother which were found in a garden of the neighborhood. The track led to a mass grave with eight bodies where my mother could identify her husband and another man who used to live in our house. By April 22, 1945, the Red Army had crossed the city limits of Berlin at several points. Although he was a civilian and not a member of the Nazi Party, my father and other civilians were executed by soldiers of the Red Army. The people who witnessed the executions were taken into Soviet captivity, and it was not possible to obtain details of the exact circumstances of my father's death.

On the other hand, the Soviet Intelligence Service was on the look-out for scientists to collaborate on research programs and had already inquired about the whereabouts of my father. The first edition of the *Wechselstromschaltungen* [22] was reprinted 1946 in the United States as a prize of victory.

During the last two years of the war, Cauer was unable to publish any results. Thanks to the energy of my mother and the kind editorial help of Dr. E. Glowatzki, Dr. G. E. Knausenberger, Dr. W. Klein, and Dr. F. Pelz, some papers were published posthumously [23], [24].

3 Cauer's Program for Network Synthesis

This section focuses on aspects of Cauer's scientific work that might be interesting from the viewpoint of general network and systems theory. For a complete list of Cauer's publications see [7] or [23].

In his 1926 doctoral thesis [8], Cauer was already sketching a complete program for network synthesis as a solution to the *inverse* problem of circuit analysis: Given the external behavior of a linear passive one-port in terms of a driving-point impedance as a prescribed function of frequency, how does one find internally passive realizations for this 'black-box'? He then shows that the synthesis problem requires the systematic solution of three main issues concerning the *realizability*, *approximation*, and *realization* of a given impedance (voltage/current transfer function) $Z(\lambda)$ as a function of the complex frequency parameter $\lambda = \sigma + \mathrm{j}\,\omega$. At the time 'passive realization' was clearly a synonym for reciprocal circuits consisting of resistors, inductors (possibly magnetically coupled), and capacitors with the positive element values R, L and C.

Cauer had a firm education in mathematical physics, and was well acquainted with Lagrangian-style analytical mechanics, particularly in terms of the treatises of E. J. Routh and E. T. Whittaker on the dynamics of rigid bodies. With a view to investigating realizability conditions, he starts with the symmetric $n \times n$ loop matrix of a generic passive n-mesh circuit (cf. Fig. 3)

$$\boldsymbol{A} = \lambda^2 \boldsymbol{L} + \lambda \boldsymbol{R} + \boldsymbol{D}, \tag{1}$$

where $\boldsymbol{R}$, $\boldsymbol{L}$, and $\boldsymbol{D}$ are positive definite matrices of resistance, inductance, and elastance (reciprocal capacitance), respectively. The pertaining quadratic forms correspond to the rate of dissipation of energy in heat and the stored electromagnetic and electrostatic energies. Assuming the external accessible port to belong to the first mesh, the input impedance is calculated by elimination of internal variables as

$$Z(\lambda) = \frac{\det \boldsymbol{A}}{\lambda\, a_{11}}, \tag{2}$$

where a_{11} is the complement of the element A_{11} in $\det \boldsymbol{A}$. Cauer emphasizes the analogy between realizable functions and the stability theory of small oscillations in classical mechanics by comparing electrical quantities to their mechanical counterparts, the Lagrange multipliers, and the classical triple of quadratic forms (kinetic, potential and dissipated energy). Moreover, he clearly points out that on the level of the generic n-mesh circuit there are absolutely no additional realizability constraints beyond positive definiteness of the three quadratic forms when ideal transformers are admitted as circuit elements. In other words, additional constraints are exclusively imposed by the topology of the circuit.

In subsequent papers [10], [11], [15] Cauer simultaneously subjects the triple of quadratic forms to a group of real affine transformations

$$\boldsymbol{T}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{T}, \quad \boldsymbol{T} = \begin{bmatrix} 1 & 0_{1,n-1} \\ T_{21} & T_{22} \end{bmatrix} \tag{3}$$

and shows that external behavior in terms of $Z(\lambda)$ in (2) is invariant! In his detailed account of Cauer's approach, N. Howitt can barely restrain his enthusiasm [34]: "Considerable has been written on electrical networks and the impedance function, but it has hardly been suspected that electrical networks formed a group with the impedance function as an absolute invariant and that it was possible to proceed in a continuous manner from one network to its equivalent network by a linear transformation of the instantaneous mesh currents and charges of the network."

On the basis of this fundamentally new concept of external equivalence of passive linear networks under transformations of internal variables, Cauer was able to state the problem of linear circuit synthesis as follows [22, p. 13], [23, p. 49]: "The previous discussions have shown that it is less important for the electrical engineer to solve given differential equations than to search for systems of differential equations (circuits) whose solutions have a desired property. With the realization of circuits with prescribed frequency characteristics in mind and in the interest of a systematic procedure, the tasks of linear network theory are formulated as follows:

(1) *Which classes of functions of λ can be realized as frequency characteristics*?

(2) *Which circuits are equivalent to each other, i.e. have the same frequency characteristics*?

(3) *How are the interpolation and approximation problems (which constitute the mathematical expression of the circuit problems) solved using functions admitted under question* (1)?"

Later on, this way of studying differential equations *indirectly* through transfer functions of black-boxes and their input-output pairs became characteristic of modern linear system theory.

3.1 Two-Element Kind Networks

In his paper *A reactance theorem* [31], R. M. Foster answers the question concerning the necessary and sufficient conditions that have to be fulfilled by a rational function $Z(\mathrm{j}\,\omega)$ if the function is to be realizable as the driving-point impedance of a lossless one-port. Furthermore, he showed that the partial fraction expansion of any such function induces a *canonical* realization, i.e. a LC circuit with the minimum number of reactances. In his dissertation [8], Cauer complemented this finding with a more concise proof of the analytical properties of the reactance function $Z(\lambda)$ and by means of his celebrated canonical *ladder realizations* (obtained via Stieltjes' continued fraction expansions). Most notably, he adapted

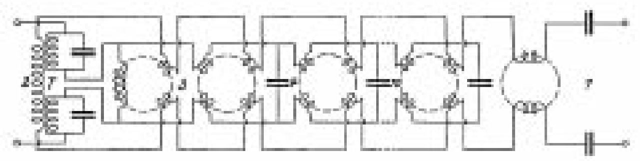

Figure 1: Symmetrized realization of a canonical reactance two-port with capacitors, coupled coils and an ideal transformer T [11].

the results for purely reactive networks to all two-element kind networks showing an isomorphism between LC, RC and RL circuits.

Based on the observation that the poles and zeros of the pertaining $Z(\lambda)$ alternate on the real or imaginary λ-axis, he established a fundamental relationship between polynomial stability tests and realization algorithms for two-element kind circuits. In [10], he emphasizes the role of a reversal of these realization algorithms: They generate parametrizations of the pertaining subclasses of positive-real functions $Z(\lambda)$ in an algebraically trivial manner without any reference to network graphs, quadratic forms or analytic function theory.

In [11], [13], [15], [20], Cauer completely solves the questions of minimal realization and equivalence for lossless reciprocal (or more generally: two-element kind) multiports with prescribed input/output behavior in two steps:

- Realization of a given n-port by partial fraction expansion of the reactance matrix $Z(\lambda)$. Due to the uniqueness of the partial fraction decomposition, this realization is canonical (minimal).
- Determination of any other externally equivalent minimal realization by equivalence transformation (3) of internal variables.

A key problem is the simultaneous principle axis transformation of two quadratic *semi*-definite forms (as opposed to the standard case where at least one form is non-degenerate). Fig. 1 shows an LC two-port obtained by transformation of the matrices $\boldsymbol{L}, \boldsymbol{D}$ in (1) to a certain normal form. One should note that in the case of LC multiports ($\boldsymbol{R} = 0_n$) it is not difficult to write down a Kalman state space realization of the impedance matrix $Z(\lambda)$ and to show that the equivalence transformation (3) contains state space equivalence as a subgroup [3].

3.2 Electrical Filters

Cauer's program was the basis of his first monograph *Siebschaltungen* (filter circuits) in 1931 [14]. This work contains a complete theoretical description of the subject matter and also contains catalogs for various types of selective filters. The book was completed in 1930/31 during Cauer's stay in the USA as a Rockefeller Fellow. In [27], S. Darlington recalls his first encounters with W. Cauer: "At Bell Laboratories a number of us first learned about Cauer's canonical circuits and his Chebyshev approximations at a conference on Cauer's proposed sale of some of his patents. It was an important event in my professional life."

The tremendous progress as compared to previous design methods was due to the introduction of *lattice realizations* for symmetric two-ports [9] and to a new approximation of prescribed filter characteristics. Cauer was the first to recognize the optimality of the Chebyshev criterion for the approximation of a constant attenuation value in a frequency band, and he later solved the Chebyshev approximation problem for filter circuits with two separate intervals on the line using elliptic functions [19].

Cauer's mathematical solution to the filter synthesis problem was apparently such a shock to the engineering community that E. A. Guillemin and R. Julia, two prominent network theorists, felt the need to explain Cauer's ideas in long and detailed papers. E. A. Guillemin [32] justified his response by "...the form of Cauer's publication, which is quite mathematical and in general not in accord with the manner in which similar material is presented in this country ..." (This explains why the paper excludes the most important and mathematically more intricate part on approximation.) R. Julia [35] confirms that Cauer's style is *beaucoup trop concise* and another French engineer laments about Cauer's first monograph *que les explications qui accompagnent ce travail soient à peu près inintelligibles, ce qui en rend l'emploi illusoire* [28].

Cauer's concept of filter synthesis was extended between 1937 and 1939 to a general systematic theory of insertion loss filter design, whereby Bader, Cauer, Cocci, Darlington, Norton and Piloty were the main contributors (an interesting discussion of the history of this subject is contained in [2], [36]). The main findings on filter synthesis are included in Cauer's second monograph *Theorie der linearen Wechselstromschaltungen* [22] (long delayed but finally published in 1941) and one can say that Cauer more or less completed his program with regard to filter design within 15 years. Indeed, in a 1958 survey on recent developments in filter theory, V. Belevitch concludes, "There has been no essentially novel theoretical progress in filter theory since 1939" [1].

3.3 General Passive Multiports

As long ago as his 1926 dissertation, Cauer uses a (now) standard passivity argument to prove that boundedness of transients in an electrical circuit imposes the fundamental *necessary* realizability condition

$$\mathrm{Re}Z(\sigma + \mathrm{j}\,\omega) > 0, \quad \forall\,\sigma > 0.$$

for any impedance function $Z(\lambda)$ at 'complex frequencies' $\lambda = \sigma + \mathrm{j}\,\omega$. With regard to *rational* impedance functions, he studies the properties of elementary two-mesh circuits under the additional constraint that no ideal transformers are admitted. Unfortunately, the journal paper [8] is only a shortened version of Cauer's dissertation (all the original copies of Cauer's thesis seem to have disappeared). It excludes most notably the chapters on *nonrational* network functions and infinite circuits. A few years later, however, Cauer emphasizes the role of nonrational impedances in an engineering exposition of his main ideas [11]: “If we do not restrict ourselves to a fixed finite number n of independent meshes but admit infinite networks, one arrives at a very simple and complete answer to the question: What is the general analytic character of a function that may be approximated with any requested precision by the driving point impedance of a finite 2-pole circuit?

> *The only rational functions of λ that are realizable as impedances of 2-pole networks have to be analytic in the right halfplane $\mathrm{Re}(\lambda) > 0$, have a positive real part in $\mathrm{Re}(\lambda) > 0$, and take on real values on the real axis.*

These characteristics are an immediate consequence of the representation of these functions as the Poisson integral

$$Z = \lambda \left[C + \int_0^\infty \frac{\mathrm{d}\psi(x)}{\lambda^2 + x} \right], \qquad (4)$$

where $C \geq 0$, ψ is a monotonically increasing function, and the integral has to be taken in Stieltjes' sense. When approximating the integral by a finite sum, we get a rational function in λ that has an immediate realization as an electric circuit. [...] In this way, we obtain an *arbitrary close approximation for Z* not only in the interior of the right half plane, but also on the boundary, i.e., for purely imaginary $\lambda = \mathrm{j}\,\omega$. When a complex impedance Z is represented graphically as a function of the frequency ω, $\psi(x)$ can only be determined on the basis of the *real* part $\mathrm{Re}Z(\mathrm{j}\,\omega)$; it can be an *arbitrary, piecewise linear non-negative* function of frequency. The point is, given a preassigned real part, *the imaginary part cannot be chosen at will* if Z has to be realizable by a 2-pole circuit.”

In essence, long before algebraic realization theory for rational impedances was producing satisfactory results of sufficient generality, Cauer obtained a complete characterization of the realizability class by means of analytic function theory (later, for this class the term *positive-real* (abbrev. PR) was coined in O. Brune's thesis [5]). This included (i) the (Hilbert) integral relation between real and imaginary parts of characteristic network functions as a limiting case of (4), (ii) results on their *rational* interpolation and approximation, and (iii) the relationship to function theoretical fundamentals such as Schwarz' lemma, cross ratios, non-Euclidian metrics and their contraction or invariance under mappings induced by passive or lossless circuits (cf. [10], [17]–[19] and Fig. 2).

On the algebraic realization side, Cauer investigated the internal structure and equivalence of multiports on the basis of (1)–(3) (cf. [15], [16], [18], and Fig. 3). In his habilitation thesis ‘On a problem where three positive definite quadratic forms are related to one-dimensional complexes’ [15], Cauer concentrated on the analytic properties of RLC multiports and the *algebraic-geometric* aspects of their canonical representation (i) in terms of rational matrices that are generated by three positive quadratic forms in n variables and (ii) in terms of their assignment to a one-dimensional cell complex

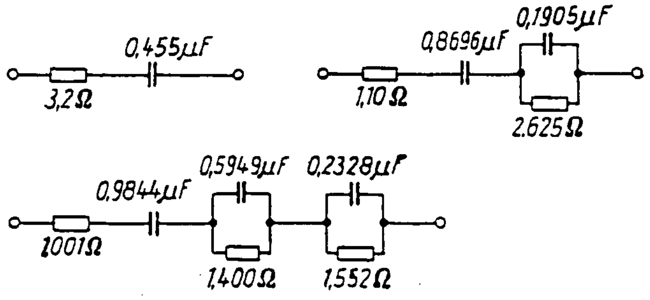

Figure 2: Three RC circuits that approximate the PR function $Z(\lambda) = 1 + \frac{1}{\lambda} + \frac{1}{\lambda+1} + \frac{2}{\lambda+2} + \frac{3}{\lambda+3}$ by successive Pick-type interpolation on the boundary $\lambda = \mathrm{j}\,\omega$ at the points $\omega_1 = 1,\ \omega_2 = 3,\ \omega_3 = 6$ (cf. [18], [23]).

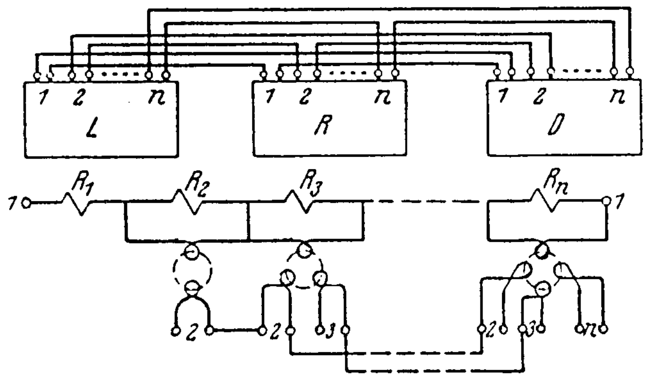

Figure 3: Canonical realization of an arbitrary finite passive multiport by three mesh-connected purely inductive, resistive and capacitive n-ports *L, R* and *D*, respectively [15]. Their internal structure is shown below for the resistive *R*-box, where minimality of the number of parameters is achieved by a special array of multi-winding transformers (obtained by reduction of the turns-ratio matrix to triangular form). External ports can be introduced by opening one or more of the external meshes.

of first Betti number n (in reference to Osvald Veblen's *Analysis Situs*). As Cauer points out, the main structural distinction between general RLC and two-element kind multiports is (i) that it is generally not possible to simultaneously diagonalize the three quadratic forms by congruence and (ii) that the occurrence of additional absolute invariants of (3) implies the non-existence of global canonical forms for the generation of all realizations. However, he showed (among other things) that the realization problem can often be split into 'smaller' ones by simultaneously transforming the quadratic forms into a common but otherwise arbitrary block-diagonal structure.

Despite all this fundamental progress on the questions of realizability, realization and equivalence, a decisive gap in synthesis theory remained until Cauer suggested and supervised the doctoral thesis of O. Brune at MIT in 1930/31. Brune provided the long-unresolved proof that the PR property is not only a necessary but also sufficient condition for a rational function to have a physical realization, i.e.

- in the form of a *finite* network with positive values of network elements $R,\ L,\ C$ (or a positive definite $\boldsymbol{L}$-matrix in case of coupled coils)
- without *ideal* transformers.

Remarkably, he showed with the help of his famous continued fraction (involving the 'Brune cycle') that in the case of scalar PR functions there are no additional topological constraints to ensure realizability without ideal transformers.

It was clear at the time that Brune's continued fraction can be extended to arbitrary complex zeros of transmission, while the Brune cycle produces only purely imaginary transmision zeros. In 1937, E. L. Norton (Bell Laboratories) published a paper on constant resistance band-separation filter pairs with only one resistor at the output terminals of each filter. This provoked the question of the minimum number of resistors and the question of the relation between Norton's cascade circuits and the Brune process – the most fundamental and penetrating structural result of classical circuit synthesis was "in the air":

> **Darlington's Theorem:** Any positive real function $Z(\lambda)$ admits a realization as the input impedance of a lossless (frequency-dependent) two-port terminated in a positive (frequency-independent) resistor R.

In other words, the set of PR rational functions $Z(\lambda)$ admits a linear fractional parametrization

$$Z(\lambda) = \frac{A(\lambda)R + B(\lambda)}{C(\lambda)R + D(\lambda)}, \quad \mathbf{T}(\lambda) = \begin{bmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{bmatrix}, \quad (5)$$

where $\mathbf{T}(\lambda)$ sweeps through the set of rational chain matrices of lossless two-ports. The numerous applications of this *lossless embedding* of passive impedances stems from its canonical separation of thermodynamical aspects (dissipation of energy) and Hamiltonian concepts (storage of energy, dynamics, holonomic constraints or frequency dependence) within the class of linear passive systems. The theorem was apparently devised by W. Cauer [21], G. Cocci [25], and S. Darlington [26], all independently of the other. Cauer's proof seems to be not only the shortest (it requires only half a printed page [21, p. 232]) but also the most elegant and forward-looking. Instead of dealing with PR impedances $Z(\lambda)$, Cauer applies a bilinear transform

$$T(\lambda) = \frac{Z(\lambda) + 1}{Z(\lambda) - 1} := \frac{g(\lambda)}{h(\lambda)}, \quad (6)$$

and discusses all rational solutions of a (spectral) factorization of the rational (and for $\lambda^2 = -\omega^2$ positive) function

$$T(\lambda)T(-\lambda) - 1 := \frac{f(\lambda)f(-\lambda)}{h(\lambda)h(-\lambda)}. \quad (7)$$

The polynomials f, g, h, in which $g(\lambda)$ is Hurwitz, are called *Betriebskenngrößen* by Cauer because they are the key quantities in the design of reactance filters whose preassigned characteristics are specified in terms of insertion loss (*Betriebsdämpfung*). Cauer's polynomials f, g, h still survive (in the very same notation!) in Belevitch's celebrated canonic form of the scattering matrix of a real lossless twoport [3, p. 428 (note 8)].

4 The Reception of Cauer's Scientific Work

Although the significance of Wilhelm Cauer's scientific work seems to be well known to many network theorists and researchers in certain fields of applied mathematics and control theory, this does not seem to be the case in present-day journals, monographs and textbooks dealing with these research areas. Another question worth investigating is whether the significance of Cauer's work was recognized by his contemporare's, and so we shall be discussing certain aspects of how the scientific community regarded his work from 1926 until 1945. We will start with some remarks about the reception of Cauer's research after his early death.

It should be mentioned that although Cauer was educated as a physicist in 1926 (diploma in technical physics) and habilitated in 1928 at the mathematical department of the famous university of Göttingen he quite seldom received attention within these academic areas.

Most network theorists link Cauer's name to a certain type of electrical filters, but the details of the deep mathematical background of filters and systems Cauer established are unknown to most electrical engineers, with the exception of a few specialists. In order to illustrate the seemingly weak influence that Cauer's work has had on present research in network and systems theory, we shall now quote several authors in this field. In the 13th edition (1975) of Hendrik W. Bode's monograph *Network Analysis and Feedback Amplifier Design* (first edition published 1945) we find the following remarks [4]: "However, special mention should be made of K.W. Wagner and W. Cauer, two Germans whose important contributions were slow to diffuse outside Germany because of the accidental intervention of World Wars I and II. [...] The enlarged second edition, edited by W. Klein and F. Pelz and translated into English by G. E. Knausenberger and J. W. Warfield, appeared in 1958 and Cauer's work became widely known at that time." Nevertheless, e.g. I. M. Horowitz's monograph *Synthesis of Feedback Systems* [33], published in 1963, uses many of Cauer's concepts but does not mention Cauer himself.

Cauer's name is also difficult to find in mathematical literature. Norbert Wiener, who met Cauer in Göttingen in 1927, and later at MIT in 1930/1931 where Cauer was a Rockefeller Fellow, refers to him briefly in his autobiography as 'Richard Cauer'. It is of interest to note in this respect that both Wiener and his Ph.D. student Y. W. Lee discussed the problem of synthesis of electrical filters around 1930/1932 using Cauer's ideas, among others. Cauer himself, however, explained Wiener's and Lee's approach in his paper and in his book in detail. Therefore, in order to gain a clearer idea of the way Cauer's work was regarded between 1928 and

1945, it is worth while to refer to the material contained in his estate.

It is not known why Cauer started to work on electrical filters, but in March 21, 1926, he wrote a letter to R. Foster, who had published his famous *Reactance Theorem* paper in 1924 and who was now working as a research scientist at the department of development and research at Bell System in New York. This paper had a considerable influence on Cauer's work. Foster replied fairly promptly with a letter dated April 8, 1926, in which he explained some aspects of their common research interests. For his part, S. Darlington mentions in his autobiographical notes that Foster referred to corresponding with Cauer in 1924 and 1926 in a phone call he made with Darlington in 1983, and Cauer's estate includes a letter written to him by Foster in August 1939, proving that they were in contact until that year.

Cauer had also contacts with mathematicians who worked in his field of interests. For example, the estate contains a postcard from Caratheodory, who was very active in the theory of complex functions. As already mentioned, Hamel and Wagner acted as the academic supervisors for Cauer's 1926 thesis, and a close connection was maintained with Hamel for many years, as the correspondence in Cauer's estate proves. Once Cauer began working at Courant's Institute of Mathematics at Göttingen University, he came into contact with many mathematicians working in related areas. Issai Schur emphasized in a letter the significance of the charge to Göttingen for Cauer's career. It was also around this time that his rich correspondence with the mathematician G. Herglotz began, which only ended in late 1944, shortly before Cauer's death. His estate also contains correspondence with the mathematician G. Pick and the graph theorist D. König from Budapest. In 1929, Cauer used his contact to N. Wiener to ask V. Bush of MIT for help with a Rockefeller grant. At this time, he was interested in mathematical machines for solving (network) determinants, which was one of Bush's main research areas.

During his Rockefeller fellowship in 1930/31, Cauer met many researchers and had strong contacts with the research staff at Bell Laboratories (Bode, Campbell, Darlington, Foster, Zobel, and others). At MIT he supervised Brune's thesis on the electrical realization of PR functions. In a footnote on the first page of his celebrated paper [6], Brune acknowledges the stimulus provided by "Dr. W. Cauer who suggested this research", and several letters in Cauer's estate illustrate the close connection between the two men.

Back in Germany, Cauer contacted other industrial companies in France and England with regard to his patents, corresponding for example with A.C. Bartlett of the General Electric Company in Wembley, and Roger Julia of Lignes Telegraph Telephone in Paris. The personal nature of these contacts was emphasized by the letter to Cauer from Bartlett of March 27, 1933, which is still in existence today and which states: "Many thanks for the reprint on Poisson Integral which you have just sent me. I am afraid I am a very bad correspondent and must also thank you for the New Years Card that you and Mrs. Cauer so kindly sent me with the etching of the Jacobikirche. I congratulate you in your new son . . ." (The son is one of the authors of this paper.) As for Cauer's contact with Julia (a brother of the mathematician Gaston Julia), the latter published a 70-page paper in 1935 with the title *Sur la Théorie des Filtres de W. Cauer*. Julia's paper is certainly the most profound treatise on Cauer's ideas on filter design and reveals excellent knowledge of the field.

A few years earlier, E.A. Guillemin of MIT, formerly a Ph.D. student of the famous physicist A. Sommerfeld in Munich, published a 60-page paper about Cauer's ideas, *A Recent Contribution to the Design of Electrical Filter Networks*. In his introductory remarks to the paper, Guillemin writes [32]: "Early last year a new method for design of electrical wave filters was published by W. Cauer, a German engineer who is well known in the field of network synthesis for various notable contributions". Before publishing this paper, Guillemin wrote a letter to Cauer (December 2, 1932) in Göttingen: "I have just recently had occasion to study more thoroughly your publication entitled 'Siebschaltungen' and am delighted with your method of attack on this problem. [. . .] Your method is the first which elegantly solves this very important problem. [. . .] I have, therefore, decided to publish an article here in which I shall compare your method of design with that of O.J. Zobel for the purpose of demonstrating the advantages which may be had when using your method, for, up to the present time, I believe that it has not received the attention it deserves." Of course, Guillemin was able to read German texts. Oddly enough, in the first issue of IRE Transactions on Circuit Theory, founded in 1952, the same Guillemin extensively discusses the question 'What is network synthesis?', but neither refers to Cauer's program nor mentions his name.

Contacts with scientists in the USA, France, England and elsewhere are documented by many letters in the estate, such as the correspondence with Darlington and Foster, which lasted until the end of 1939. In a letter dated April 26, 1939, Cauer responded to Darlington (in German!) "It was with great interest that I examined your D.R.P. 673 336 patent specification. This is because without knowing your patent, I have used the same elliptic function formulae in an essay titled *Frequenzweichen konstanten Betriebswiderstandes* that will be appearing shortly in the *Elektrische Nachrichtentechnik* E.N.T." Cauer added a remark referring to Darlington's patent in the final version of this paper. The estate also includes a letter from Foster of August 8, 1939, in which he thanks Cauer for sending him three papers and promising to send him the *Collected Papers of G. A. Campbell* in return. The correspondence with many other network theorists that is also contained in Cauer's estate proves that his work and results were well known to many researchers in this field, and that Cauer himself had close personal contacts with these colleagues.

In contrast to this substantial correspondence with foreign researchers, Cauer's correspondence with German colleagues working in the field of electrical engineering is less voluminous. Very few letters go deeply into Cauer's research interests, and for many years they were restricted to narrow-minded discussions of simple network problems (e.g., with H. Barkhausen of the University of Dresden in August 1934) or priority problems (with K. Küpfmüller, director at Siemens & Halske in Berlin, and professor at the Technical University of Berlin, or with A. Jaumann, a member of the staff at Siemens).

During the difficult years at Göttingen University after 1933, and later on after he had left this establishment, Cauer found little support from his colleagues in gaining a chair in applied mathematics or theoretical electrical engineering. This can also be documented by the fact that R. Feldtkeller in his bibliographic collection of papers on cable communications engineering cites only one paper vaguely to do with Cauer, namely one drawn up by the latter's assistant Glowatzki [29]. In a second part, which covers the period from 1936 to 1941 Feldtkeller only cites a single paper by Cauer. Moreover, the first edition of his famous treatise on *Vierpoltheorie*, which was published in 1937, Feldtkeller avoids quoting Cauer‘s name though he cites several of Cauer‘s results [30].

H. Piloty, who became a professor at the Technical University of Munich in 1936 and who started detailed studies in network theory based on Cauer's approach, does not indicate much enthusiasm for Cauer's work in the correspondence with the latter. After the publication of Cauer's book *Wechselstromschaltungen*, Piloty wrote him a long letter dated December 15, 1941, containing many critical comments in order to substantiate his claims to several research results. Cauer replied with equally strong claims.

After 1935, Cauer abandoned all hope of getting a chair and joined Mix & Genest in Berlin instead. He sought to become active in the German Society of Electrical Engineers (VDE), but after proposals he made for talks on his research interests had been repeatedly rejected and after a heated discussion with Wagner in 1942 on the topic of the latter's disinterest in supporting Cauer's scientific career, he left the VDE. Moreover, although Cauer was a well-respected scientist in network theory, he never gained a chair, that he so much deserved. Beside the problems mentioned in Section 2, Cauer was handicapped in that he was educated as a mathematician and thus always took a mathematical approach. He was not an electrical engineer, although his knowledge of electrical circuits was both broad and excellent. He had a similar problem to Hermann Weyl, his former colleague in Göttingen, as expressed by the latter in his preface to the first German edition of *Gruppentheorie und Quantenmechanik*: "In this drama of mathematics and physics, two disciplines that fertilize each other in secret and misjudge and deny each other in the open, I simply cannot desist from playing the role of envoy (and an undesired one at that, as I have frequently had to experience)."

5 Conclusion

Though the life and work of Wilhelm Cauer were fragmentary in some respects, and despite his early death in 1945, "there is no question in my mind that he was not only the first, but also the most distinguished and creative of all modern network theorists" (D. C. Youla 1994 in a letter to the third author).

Although he opened a new boundary field between mathematics and electrical engineering, Cauer had to either stand up for his ideas in a climate of disinterest or defend them against harsh criticisms from both disciplines. Nevertheless, he succeeded in realizing his scientific program with regard to the synthesis of electrical filters to such an extent that as late as 1958 V. Belevitch still referred to Cauer's 1941 treatise as "the most complete single reference source" in the field [1]. Cauer‘s success in resolving engineering design problems is also demonstrated by the fact that he was able to subsidize his assistant E. Glowatzki in Göttingen with the help of the patent royalties he received from such firms as Bell Company.

Section 3 of this paper contains a discussion by Rainer Pauli of the main aspects of Cauer's scientific work on network synthesis, particularly with regard to findings that are of interest in terms of general network and systems theory. Sections 2 and 4 comprise an examination of Wilhelm Cauer's personality and career by Emil Cauer and Wolfgang Mathis, based on details provided by Cauer's family as well as letters and documents from his scientific estate.

It is naturally spurious to wonder how Wilhelm Cauer's work would have continued had the scientist and mathematician survived the Second World War. Nevertheless, of one thing we can be sure: his fruitful vision of systems synthesis – enabled as it was by his interdisciplinary approach – would have had an enormous influence on the development of network and systems theory. Furthermore, his life also demonstrates that he had the diligence and ability to translate this vision into a powerful technical concept.

The reasons why Cauer's career never benefited from his seminal scientific discoveries are manifold and complex. Nevertheless, his ideas have survived to the present day in many areas of mathematics and systems theory. Indeed, his program for the systematic *synthesis* of networks according to prescribed technical performance criteria makes him the first modern system theorist in the sense of R. E. Kalman's definition [37]:

> "System theory is not a branch of 'natural science'; it is not a study of (physical) nature. It is the study of Man‘s ingenuity; not a study of things as they are but of things to be."

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