Local Illumination, Reflection, and BRDFs

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Overview

• Radiometry and Photometry
• Definition of BRDF
• BRDF properties and common BRDFs
• Rendering equation
Radiometric Units

- Light is a form of energy – measured in Joules (J)
- Power: energy per unit time
  - Measured in Joules/sec = Watts (W)
  - Also called Radiant Flux (Φ)

Point Light Source

- Total radiant flux in Watts
- How to define angular dependence?
  - Solid angle
  \[ \text{Solid angle } d\omega = dA/r^2 \]
- Power per unit solid angle
  - Measured in Watts per steradian (W/sr)
Light Falling on a Surface

- Power per unit area – *Irradiance* (E)
  - Measured in W/m²
- Move surface away from light
  - Inverse square law: \( E \sim \frac{1}{r^2} \)
- Tilt surface away from light
  - Cosine law: \( E \sim n \cdot l \)

Light Emitted from a Surface

- Power per unit area per unit solid angle – *Radiance* (L)
  - Measured in W/m²/sr
  - *Projected area* – perpendicular to given direction

\[ L = \frac{d\Phi}{dA
d\omega} \]
Total Light Emitted from a Surface

- Radiance integrated over all directions

\[ B = \int_{\Omega} L_\omega(\theta, \phi) \cos \theta \, d\omega \]

- Called *Radiosity* (B)
  - Measured in W/m²

Radiometry vs. Photometry

- These are all physical (radiometric) units
- Don’t take perception into account
- Eye sensitive to different colors

\[ \lambda \text{ (nm)} \]

\[ 400 \text{ (blue)} \quad 700 \text{ (red)} \]
Photometric Units

- Take human perception into account
- Original unit: candle
  - Luminous intensity equal to a “standard candle”
- Today: one of the basic SI units
  - One candela (cd) is the luminous intensity of a source producing 1/683 W at 555 nm.

Radiometric and Photometric Units

<table>
<thead>
<tr>
<th>Radiometric Units</th>
<th>Photometric Units</th>
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</thead>
<tbody>
<tr>
<td>Radiant energy</td>
<td>Luminous energy</td>
</tr>
<tr>
<td>Joule (J)</td>
<td>Talbot</td>
</tr>
<tr>
<td>Radiant flux or power ($\Phi$)</td>
<td>Luminous power</td>
</tr>
<tr>
<td>Watt (W) = J / sec</td>
<td>Lumen (lm) = talbots / sec</td>
</tr>
<tr>
<td>Radiant intensity ($I$)</td>
<td>Luminous intensity</td>
</tr>
<tr>
<td>W / sr</td>
<td>Candela (cd)</td>
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<td>Irradiance ($E$)</td>
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Direct Illumination

\[ E = \frac{\Phi}{A} \]

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\[ \Phi = I \omega \]
**Direct Illumination**

\[ E = \frac{\Phi}{A} \]

\[ \Phi = I \omega \]

\[ \omega = \frac{A(\hat{n} \cdot \hat{i})}{r^2} \]

\[ E = \frac{I(\hat{n} \cdot \hat{i})}{r^2} \]

**Imaging**

Surface  
Lens  
Image Plane (film, CCD)
Imaging

\[ I = L A_{surf} \]

\[ \Phi = L A_{surf} \frac{A_{aperture}}{d_{surf}^2} \]

\[ \Phi = L A_{surf} \frac{A_{img}}{A_{surf}} \]
Imaging

\[ I = LA_{surf} \]
\[ \Phi = LA_{surf} \frac{A_{aperture}}{d_{surf}^2} \]
\[ E = \frac{\Phi}{A_{img}} \]

\[ E = L \frac{A_{aperture} A_{surf}}{d_{surf}^2 A_{img}} \]

\[ \frac{A_{surf}}{A_{img}} = \left( \frac{d_{surf}}{d_{img}} \right)^2 \]

\[ E = L \frac{A_{aperture}}{d_{img}^2} \]

- Punch line: cameras “see” radiance

Surface Reflectance – BRDF

- Bidirectional Reflectance Distribution Function

\[ f_r(\omega_i \rightarrow \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)} \]

- 4-dimensional function: also written as

\[ f_r(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{dL_o(\theta_o, \phi_o)}{dE_i(\theta_i, \phi_i)} \]

(the symbol \( \rho \) is also used sometimes)
Defining Surface Reflectance

- Why is BRDF defined in this way?
- Key point: BRDF is a differential quantity, so limit must exist

\[ f_r = \frac{\Phi_{\text{det}}}{\Phi_{\text{src}}} \]

Definition of BRDF

- First attempt:
Definition of BRDF

• Should $f_r$ vary with $\omega_{src}$? No.

Source
$\Phi_{src}$
$\omega_{src}$

Detector
$\Phi_{det}$
$\omega_{det}$

Definition of BRDF

• Should $f_r$ vary with $\omega_{det}$? Yes.

Source
$\Phi_{src}$
$\omega_{src}$

Detector
$\Phi_{det}$
$\omega_{det}$
Definition of BRDF

- Thus,

\[ f_r = \frac{\Phi_{\text{det}}/\omega_{\text{det}}}{\Phi_{\text{src}}} \]

Definition of BRDF

- What about surface area?
  \( f_r \) must be independent of surface area
Definition of BRDF

\[ f_r = \frac{\Phi_{det} / (\omega_{det} \cdot dA)}{\Phi_{src} / dA} = \frac{L}{E} \]

Properties of the BRDF

- Energy conservation:
  \[ \int_{\Omega} f_r(\theta_i, \phi_i, \theta_o, \phi_o) \cos \theta_o d\omega_o \leq 1 \]

- Helmholtz reciprocity:
  \[ f_r(\omega_i \rightarrow \omega_o) = f_r(\omega_o \rightarrow \omega_i) \]

(not always obeyed by “BRDFs” used in graphics)
**Isotropy**

- A BRDF is isotropic if it stays the same when surface is rotated around normal

- Isotropic BRDFs are 3-dimesional functions:
  \[ f_r(\theta_i, \theta_o, \phi_i - \phi_o) \]

**Anisotropy**

- Anisotropic BRDFs do depend on surface rotation
Diffuse

• The simplest BRDF is “ideal diffuse” or Lambertian: just a constant

\[ f_r(\omega_i \rightarrow \omega_o) = k_d \]

• Note: does not include \( \cos(\theta_i) \)
  - Remember definition of irradiance

Diffuse BRDF

• Assume BRDF reflects a fraction \( \rho \) of light

\[
\int_{\Omega} f_{r,\text{Lambertian}}(\omega_i \rightarrow \omega_o) \cos \theta_o \, d\omega_o = \rho \\
\int_{\theta_o[0, \frac{\pi}{2}]} k_d \cos \theta_o \sin \theta_o \, d\theta_o \, d\varphi_o = \rho \\
2\pi k_d \int_{\theta_o[0, \frac{\pi}{2}]} \sin \theta_o \cos \theta_o \, d\theta_o = \rho \\
\pi k_d = \rho \\
\therefore f_{r,\text{Lambertian}} = \frac{\rho}{\pi}
\]

• The quantity \( \rho \) is called the albedo
Ideal Mirror

• All light incident from one direction is reflected into another

• BRDF is zero everywhere except where

\[ \theta_o = \theta_i \]
\[ \phi_o = \phi_i + \pi \]

Ideal Mirror

• To conserve energy,

\[ \int_{\Omega} f_{r,Mirror}(\omega_i \rightarrow \omega_o) \cos \theta_o d\omega_o = \rho \]

• So, BRDF is a delta function at direction of ideal mirror reflection

\[ f_{r,Mirror} = \frac{\delta(\theta_i - \theta_o) \delta(\phi_i - \phi_o)}{\cos(\theta_i)} \]
**Glossy Reflection**

- Non-ideal specular reflection
- Most light reflected *near* ideal mirror direction

**Phong BRDF**

- Phenomenological model for glossy reflection
  \[ f_{r, \text{Phong}} = k_s (\hat{l} \cdot \hat{r})^n \]
  - \( l \) is a vector to the light source
  - \( r \) is the direction of mirror reflection
  - Exponent \( n \) determines width of specular lobe
  - Constant \( k_s \) determines size of lobe
Torrance-Sparrow BRDF

• Physically-based BRDF model
  ◦ Originally used in the physics community
  ◦ Adapted by Cook & Torrance and Blinn for graphics

\[ f_{r,T-S} = \frac{DGF}{\pi \cos \theta_i \cos \theta_o} \]

• Assume surface consists of tiny “microfacets” with mirror reflection off each

Torrance-Sparrow BRDF

• \( D \) term is distribution of microfacets (i.e., how many are pointing in each direction)

• Beckmann distribution

\[ D = \frac{e^{-[(\tan \beta)/m]^2}}{4m^2 \cos^4 \beta} \quad \beta \text{ is angle between } n \text{ and } h \]
\[ h \text{ is halfway between } l \text{ and } v \]
\[ m \text{ is “roughness” parameter} \]
Torrance-Sparrow BRDF

- $G$ term accounts for self-shadowing

$$G = \min\left(1, \frac{2(n \cdot h)(n \cdot v)}{(v \cdot h)}, \frac{2(n \cdot h)(n \cdot l)}{(v \cdot h)}\right)$$

- $F$ term is Fresnel term – reflection from an ideal smooth surface (solution of Maxwell’s equations)

- Consequence: most surfaces reflect (much) more strongly near grazing angles

Dielectric    Metal

(note behavior at Brewster’s angle)
Other BRDF Features

• BRDFs for dusty surfaces scatter light towards grazing angles

Other BRDF Features

• Retroreflection: strong reflection back towards the light source

• Can arise from bumpy diffuse surfaces

• … or from corner reflectors
BRDF Representations

- Physically-based vs. phenomenological models
- Measured data
- Desired characteristics:
  - Fast to evaluate
  - Maintain reciprocity, energy conservation
  - For global illumination: easy to importance sample

Beyond BRDFs

- So far, have assumed 4D BRDF
- Function of wavelength: 5D
- Fluorescence (absorb at one wavelength, emit at another): 6D
- Phosphorescence (absorb now, emit later): 7D
- Temporal dependence: 8D
- Spatial dependence: 10D
- Subsurface scattering: 12D
  - Polarization
  - Wave optics effects (diffraction, interference)
- …
The rendering equation is given by:

\[
L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega} f_r(x, \omega_i \rightarrow \omega_o) L_i(x, \omega_i) \cos \theta_i \, d\omega
\]

- Outgoing radiance
- Emitted radiance
- BRDF
- Irradiance

- Originally expressed by [Kajiya 1986] as

\[
I(x' \rightarrow x'') = I_e(x' \rightarrow x'') + G(x', x'') \int \frac{f_r(x \rightarrow x' \rightarrow x'')}{s} I(x \rightarrow x') V(x, x') \, dA
\]
Rendering Equation

- Originally expressed by [Kajiya 1986] as
  \[
  I(x' \rightarrow x'') = I_e(x' \rightarrow x'') + \int_S G(x', x'') f_e(x \rightarrow x' \rightarrow x'') I(x \rightarrow x') V(x, x') dA
  \]
- Integral is over all points in the scene
- \( G(x, x') \) is a geometry term:
  \[
  G(x, x') = \frac{\cos \theta'_e \cos \theta_e}{\|x - x\|^2}
  \]

Rendering Equation

- Originally expressed by [Kajiya 1986] as
  \[
  I(x' \rightarrow x'') = I_e(x' \rightarrow x'') + \int_S G(x', x'') f_e(x \rightarrow x' \rightarrow x'') I(x \rightarrow x') V(x, x') dA
  \]
- Integral is over all points in the scene
- \( V(x, x') \) is a visibility term and is either 0 or 1
Rendering Equation

• Originally expressed by [Kajiya 1986] as
\[ I(x' \rightarrow x'') = I_e(x' \rightarrow x'') + \]
\[ G(x', x'') \int f_s(x \rightarrow x' \rightarrow x'') I(x \rightarrow x') V(x, x') dA \]

• Integral is over all points in the scene
• \( I(x \rightarrow x') \) is the two-point transport intensity:

\[ I(x \rightarrow x') = L(x, \omega) G(x, x') dA dA' \]

(note: this is not the same \( I \) we’ve seen before…)

Rendering Equation

• Next 3-4 weeks in the course: ways to solve the rendering equation