

**Computer Science 341**  
**Discrete Mathematics**

Problem Session 6  
November 4, 2002

Problem 1

Prove that the complement of a disconnected graph  $G$  is connected. (The *complement* of a graph  $G = (V, E)$  is the graph  $(V, \binom{V}{2} \setminus E)$ .)

Problem 2

Show that a simple graph  $G$  contains a  $K_3$  (i.e., a triangle) if and only if there exist indices  $i$  and  $j$  such that both  $A_G$  (the adjacency matrix of  $G$ ) and  $A_G^2$  have a nonzero in entry  $(i, j)$ .

Problem 3

Find an example of an asymmetric graph with at least 2 vertices (and prove that it is indeed asymmetric). Recall that a graph is *asymmetric* if its only automorphism is the identity mapping (each vertex is mapped to itself). An *automorphism* is an isomorphism of  $G$  and itself.

Problem 4

A graph  $G$  is called *k-regular* if all its vertices have degree exactly  $k$ . Determine all pairs  $(k, n)$  such that there exists a  $k$ -regular graph on  $n$  vertices.

Problem 5

A mouse eats his way through a  $3 \times 3 \times 3$  cube of cheese by tunnelling through all of the 27  $1 \times 1 \times 1$  subcubes. If he starts at one of the corner subcubes and always moves into an uneaten adjacent subcube, can he finish at the center of the cube? Assume that he can tunnel through walls but not edges or corners.

Problem 6

A graph  $G$  has  $n$  vertices and  $k$  components. What is the maximum possible number of edges it can have?