**Computer Science 341**
**Discrete Mathematics**

Problem Session 3
October 7, 2002

Problem 1

In how many ways can one select $n$ letters from an unlimited supply of A's, B's, and C's so that there are an even number of A's (zero counts as an even number)?

Problem 2

An *ordered partition* of an integer $n$ is an *ordered* sequence of $(a_1, \ldots, a_k)$ such that $a_i > 0$ for each $i$ and $a_1 + \cdots + a_k = n$.

(a) How many ordered partitions of $n$ are there?

(b) Using generating functions, find the number of ordered partitions into an even number of parts, that is, the number of ordered partitions $(a_1, \ldots, a_k)$ where $k$ is even.

Problem 3

Find the ordinary generating function of the sequence $a_n = (n+1)(n+2)(2n+3)$.

Problem 4

Find the ordinary generating function with coefficients $a_k$ equal to the number of ways we can distribute $k$ pieces of candy to $n$ children such that no child gets more than $m$ pieces.

Problem 5

Find a closed form expression for the ordinary generating functions with coefficients

(i) $a_n = \sum_{k=0}^{n} \binom{n+k}{2k}$

(ii) $b_n = \sum_{k=0}^{n-1} \binom{n+k}{2k+1}$

Problem 6

In how many ways can $3r$ balls be selected from $2r$ identical red balls, $2r$ identical blue balls, and $2r$ identical white balls? (Selections are distinct if they do not have the same number of balls of each color.)