Problem Session 4
October 14, 2002

Problem 1

Let a_n be the number of ways to place n distinct people in 5 different rooms such that there are 2 or 3 people in each of the first two rooms. Find the exponential generating function corresponding to the sequence $a_0, a_1, \ldots, a_n, \ldots$. Then find a closed form expression for a_n.

Problem 2

There are n people waiting in a line, such that each person is able to see the r people waiting on line directly ahead of himself. Find a simple recurrence relation for $P(k, n)$, which represents the number of different ways of choosing k people from the line so that no one of them is able to see another. (Do NOT try to solve the recurrence relation!) Be sure to include appropriate boundary conditions.

Problem 3

Find a recurrence relation for a_n, the number of n-digit ternary sequences without the subsequence 012.

Problem 4

Solve the following recurrence relations:

    a. $a_n = 3a_{n-1} + 4a_{n-2}, a_0 = a_1 = 1$

    b. $a_n = 4a_{n-1} - 4a_{n-2}, a_0 = 5, a_1 = 14$

    c. $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}, a_0 = a_1 = 1, a_2 = 2$

    d. $a_n = 3a_{n-2} + 2a_{n-3}, a_0 = a_1 = 1, a_2 = 2$