Computer Science 341
Discrete Mathematics

Homework 9
Due at the beginning of class on Wed, Nov 27, 2002

Collaboration Policy: You may collaborate in groups of at most 3 students. These groups must be disjoint and discussion across groups is not allowed. Collaboration is limited to discussion of ideas only, and you should write up the solutions entirely on your own and list your collaborators.

Problem 1
Let G be a connected planar graph which satisfies the following conditions for some $p \leq q$:

- every node has degree p or degree q,
- p nodes have degree q and q nodes have degree p,
- every face is a p-gon or a q-gon,
- p faces are q-gons and q faces are p-gons.

Prove that there is only one possible pair (p,q).

Problem 2
Define $\gamma(G)$ to be the smallest number of colors necessary to color the *edges* of a graph G such that edges which share a common vertex are of different colors.
Let K_n denote the n node complete graph. Show that for every positive integer k:

$$\gamma(K_{2k}) = 2k - 1$$

and

$$\gamma(K_{2k+1}) = 2k + 1$$

Problem 3
Consider an infinite graph G defined as follows: The set of vertices of G is $\{(a, b)|$ a and b are positive integers$\}$; every vertex (a, b) is adjacent to all vertices $(a + b, 1)$, $(a + b, 2)$,...,$(a + b, n)$,..., and thus to all points having positive integer coordinates on the line $x = a + b$. Show that (1) G does not contain any triangles, and (2) $\chi(G) = \infty$.

Problem 4
Prove that if a maximal planar graph is 3-colorable, then it is Eulerian.