Computer Science 341
Discrete Mathematics

Homework 8
Due in class on Wed, Nov 20, 2002
Note: homeworks will be collected at the beginning of class

No collaboration is permitted for this homework.

Problem 1
Consider a connected graph $G(V, E)$ with a non-negative weight $w(e)$ for each edge $e \in E$. Suppose we are given a spanning tree T of G. Consider the *swap* operation which modifies T by adding an edge not in T and deleting an edge on the resulting cycle. Suppose we apply this operation on T (if it reduces the cost) until it is no longer possible to lower the cost. Prove that the resulting tree T is now a minimum spanning tree for G.

Problem 2
Give a drawing of K_7 on the torus. Use the representation of drawings on the torus discussed in class and illustrated on page 173 of the text. (You may give a drawing of K_6 on the torus for half credit). *Note:* Please be neat ! Untidy drawings will be penalized.

Problem 3
Let G be a planar Eulerian graph. Consider some planar drawing of G. Show that there exists a closed Eulerian tour that never crosses itself in the considered drawing (it may touch itself at vertices but it never "crosses over to the other side").

Problem 4
Consider an arbitrary drawing (with possibly intersecting arcs) of the complete graph K_n on the plane. Prove that at least $\frac{1}{5}\binom{n}{4}$ pairs of edges have to cross. (*Hint*: Use the fact that K_5 is not planar.)