

**Computer Science 341**  
**Discrete Mathematics**

Homework 5

Due by 4:30pm, Fri, Oct 25, 2002

**Collaboration Policy:** You may collaborate in groups of at most 3 students. These groups must be disjoint and discussion across groups is not allowed. Collaboration is limited to discussion of ideas only, and you should write up the solutions entirely on your own and list your collaborators.

Problem 1

Solve the following recurrence relations using the “particular + homogeneous” solution method:

a.  $a_n + 2^n = 3a_{n-1} - 2a_{n-2} + 5$ , with initial values  $a_0 = 5$ ,  $a_1 = -1$ .

b.  $a_n + 2a_{n-2} + a_{n-4} = 0$ , with initial values  $a_0 = 0$ ,  $a_1 = 1$ ,  $a_2 = 2$ ,  $a_3 = 3$ .

Problem 2

Solve the following recurrence relations:

a.  $T(n) = 4 \cdot T(n/2) + n(\log n)^2$ ,  $T(1) = 1$

b.  $T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n$ ,  $T(2) = 1$

Problem 3

Find the number of ways to arrange  $n$  distinct people around round tables such that the number of people sitting at each table is even. The tables are not numbered. (Note: there can be no table with 0 people).

Problem 4

Find the number of ways of tiling a  $3 \times n$  board with  $1 \times 2$  tiles such that the whole board is completely covered. Note that no square should be covered more than once.

Problem 5

Let  $S_m(n) = \sum_{k=0}^{n-1} k^m$ . In this exercise, we will derive a technique to express  $S_m(n)$  as a polynomial in  $n$ .

a. Define the sequence  $B_n$  by the following equations:

$$\begin{aligned} B_0 &= 1 \\ \sum_{j=0}^m \binom{m+1}{j} B_j &= 0 \quad \forall m \geq 1 \end{aligned}$$

Find the exponential generating function  $B(x)$  for the sequence  $B_n$ .

- b. Let  $S(x, n)$  be the exponential generating function for the sequence whose  $m$ th term is  $S_m(n)$ . Find a closed form expression for  $S(x, n)$ .
- c. By relating the exponential generating functions  $B(x)$  and  $S(x, n)$ , prove that

$$S_m(n) = \frac{1}{m+1} \left( \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k} \right)$$