Computer Science 341
Discrete Mathematics

Homework 4
Due in class on Wed, Oct 16, 2002

Collaboration Policy: You may collaborate in groups of at most 3 students. These groups must be disjoint and discussion across groups is not allowed. Collaboration is limited to discussion of ideas only, and you should write up the solutions entirely on your own and list your collaborators.

Problem 1
How many non-negative integers less than $10^{(n+1)}$ have

a. an even number of 1's?

b. an even number of 0's? (Numbers cannot have leading 0's)

Problem 2
Let h_n denote the number of ways to color the squares of a 1-by-n board with the colors red, white, blue, and green in such a way that the number of squares colored red is even, and the number of squares colored white is odd. Determine the exponential generating function for the sequence $h_0, h_1, \ldots, h_n, \ldots$, and then find a simple formula for h_n.

Problem 3
Let a_n be the number of ways to pass out n pieces of candy to three children so that each child gets at least two pieces. Assume that the n pieces of candy are distinguishable.

a. What is the exponential generating function for the sequence a_n ?

b. Find an explicit formula for a_n.

Problem 4
We have n dollars. Every day we buy exactly one of the following products: pretzel (1 dollar), candy (2 dollars), ice cream (2 dollars). We continue in this way until the n dollars have been spent. What is the number B_n of distinguishable ways of spending all of the money?

Problem 5
Consider a convex n-gon (a polygon with n sides). A *chord* is a line segment between two non-adjacent vertices. Let T_n be the number of ways of selecting $n-3$ chords such that no two chords interesect (except at the vertices of the n-gon). Thus, $T_3 = 1$, $T_4 = 2$ and so on. For convenience, we define T_2 to be 1. Show that $T_n = C_{n-2}$ where C_n is the number of ways of parenthesizing a product of $n+1$ numbers (as discussed in class). (Hint: Find a recurrence relation for T_n).