Computer Science 341
Discrete Mathematics

Homework 3
Due in class on Wed, Oct 9, 2002

Collaboration Policy: You may collaborate in groups of at most 3 students. These groups must be disjoint and discussion across groups is not allowed. Collaboration is limited to discussion of ideas only, and you should write up the solutions entirely on your own and list your collaborators.

Problem 1

n married couples have to be seated at a round table with $2n$ chairs. Two arrangements are considered identical if one can be obtained from the other by a rotation around the table.

(a) How many ways can we seat the n couples if the first k couples insist that they be seated together, i.e. for each of the first k couples, the husband should be seated next to the wife ?

(b) How many ways can we seat the n couples such that no husband wife pair sits next to each other ? (Hint: use the result you get in part (a)).

Problem 2

(a) Find a closed form expression for the generating function whose n-th coefficient is a_n, where $a_0 = 0$ and $a_n = \frac{1}{n}$ for $n > 0$.

(b) Find a closed form expression for the generating function whose n-th coefficient is c_n, where $c_0 = 0$ and

$$c_n = \sum_{k=1}^{n-1} \frac{1}{k(n-k)}$$

for $n > 0$.

Problem 3

(a) Find the generating function of the sequence $a_n = (n+1)^3$.

(b) Use the method discussed in class to prove that

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Problem 4

In how many ways can 30 identical balls be distributed in 6 different boxes, such that each of the first 4 boxes receives between 3 and 7 balls?

Problem 5

(a) Show that if n is a positive integer, then

$$\binom{-1/2}{n} = \frac{\binom{2n}{n}}{(-4)^n}$$

(b) Using (a), show that the coefficient of x^n in the expansion of $(1-4x)^{-1/2}$ is $\binom{2n}{n}$ for all integers $n \geq 0$.

(c) Using (b), prove that

$$\sum_{k=0}^{n} \binom{2k}{k}\binom{2n-2k}{n-k} = 4^n$$