Computer Science 341
Discrete Mathematics

Homework 2
Due in class on Wed, Oct 2, 2002

New Collaboration Policy: **You may collaborate in groups of at most 3 students. These groups must be DISJOINT and discussion across groups is not allowed. If you have a question about the problem set, please see the TAs or the instructor. You are not supposed to discuss homework problems with students in other groups. All group members must list the members of their group and it is important that you list your group correctly. As before, discussion in a group is limited to discussion of ideas only, and you should write up the solutions entirely on your own.**

Problem 1

Sixteen different committees were generated from a group of 5 senators. It turned out that every set of 3 committees had at least one member who participated in all three.

(a) Prove that there is a member who serves in all 16 committees.

(b) Is it possible that all 16 committees have 2 members in common ?

Problem 2

How many positive integers less than 10^n (in the decimal scale) have their digits in non-decreasing order?

Problem 3

Give a formula for the number of m-digit integers in which each digit (0–9) appears at least once. (Consider integers with fewer than m digits as m-digit integers starting with zeros; for instance, 0123456789 should be considered a 10-digit integer.)

Problem 4

Consider a convex n-gon (a polygon with n sides). A *chord* is a line segment

between two non-adjacent vertices. If all chords are drawn, in how many interior points do they intersect? (Assume no three chords intersect at the same point.)

Problem 5

Show using a combinatorial interpretation, that,

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

Problem 6

Consider the summation:

$$\sum_{k=0}^{n} k\binom{n}{k}.$$

Give a combinatorial interpretation of this summation and use it to obtain a closed form expression for it.