Computer Science 341
Discrete Mathematics

Homework 10
Due at noon on Fri, Dec 6, 2002

No collaboration is permitted for this homework.

Problem 1

Consider a group of nk politicians who must be assigned to n committees. Each politician is qualified to belong to some but not necessarily all the committees. Each committee must be assigned exactly k qualified members, and each politician must be assigned to exactly one of the committees for which that politician qualifies. Give a necessary and sufficient condition on the qualifications of the politicians for the committees for this to be possible. (*Hint*: For $k = 1$ the condition of Hall's Theorem suffices.)

Problem 2

Let X be an n-element set and let $S_1, S_2, \ldots, S_n$ be subsets of X such that $|S_i \cap S_j| \leq 1$ whenever $1 \leq i \leq j \leq n$. Prove that at least one of the sets S_i has size at most $C\sqrt{n}$ for some absolute constant C (independent of n).

Problem 3

The Princeton Tigers are playing a rival team in a 2-out-of-3 series, i.e. they play games until one team wins a total of two games. The probability that the Tigers win the first game is 1/2. For subsequent games, the probability of winning depends on the outcome of the preceding game; the team is energized by victory and demoralized by defeat. If the Tigers win a game, then they have a 2/3 chance of winning the next game. On the other hand, if the team loses, they have only a 1/3 chance of winning the following game.

(a) What is the probability that the Tigers win the 2-out-of-3 series given that they win the first game ?

(b) What is the probability that the Tigers win the first game, given that they win the series ?

Problem 4

Let $S = \{1, 2, \ldots, n\}$ and let π be a random permutation on S, i.e. $\pi : S \rightarrow S$ is a random bijection from S to S For a subset A of S, let $f(A)$ be the minimum of $\pi(a)$ over all elements $a \in A$. Let A and B be two arbitrary subsets of S. Express the following in terms of A and B.

(a) $\Pr[f(A) = f(B)]$

(a) $\Pr[f(A) < f(B)]$