Computer Science 341
Discrete Mathematics

Homework 1
Due in class on Wed, September 25, 2002

Collaboration Policy: You may collaborate with no more than three other students. However, collaboration is limited to discussion of ideas only, and you should write up the solutions entirely on your own and list your collaborators.

Problem 1

Consider the following game: a pile of n stones is placed in front of two players. The players take turns removing $1, 2$ or 3 stones from the pile. The player who removes the last stone loses the game. Prove that if n gives a remainder of 1 when divided by 4 then the second player has a winning strategy, i.e. can always win the game no matter how the first player plays. Also prove that in all other cases the first player has a winning strategy.

Problem 2

Prove that the regions on the plane formed by the intersection of lines no three of which pass through the same point can be colored with 2 colors such that no two adjacent regions have the same color. Note that two regions that meet only at a single point are not considered adjacent.

Problem 3

In class, we proved the following statement using the pigeonhole principle: Given a set of $n + 1$ positive integers none exceeding $2n$, there is at least one integer in this set that divides another integer in the set. Give a proof of this using mathematical induction.

Problem 4

Suppose that n teams play a round-robin tournament in which there are no tie games. Prove that, no matter what the individual game outcomes are, one can always number the teams $t_1, t_2, \ldots, t_n$, so that t_1 beat t_2, t_2 beat t_3, and so on, through t_{n-1} beat t_n.

Problem 5

Consider these two assumptions:
1. Logic is difficult, or not many students like logic.
2. If mathematics is easy, then logic is not difficult.

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

a) Mathematics is not easy, if many students like logic.

b) Not many students like logic, if mathematics is not easy.

c) Mathematics is not easy or logic is difficult.

d) Logic is not difficult or mathematics is not easy.

e) If not many students like logic, then either mathematics is not easy or logic is not difficult.