Computer Science 341
Discrete Mathematics

Problem Session 7
Mon, Nov 11, 2002

Problem 1

Show that a graph $G = (V, E)$ with n vertices and maximum degree $\lceil n/(r-1) \rceil - 2$ contains an independent set of size r.
(An independent set is a subset of V no pair of which is an edge in E.)

Problem 2

Let G be a simple graph with n vertices ($n > 3$) such that for every pair of non-adjacent vertices (v, w), it is true that $d(v) + d(w) \geq n$ ($d(v)$ and $d(w)$ denote the degrees of v and w, respectively). Prove that G has a Hamiltonian cycle. (Note: this is one version of Dirac's Theorem).

Problem 3

What is the minimum value of $|E|$ which guarantees that G has a Hamiltonian path? Prove your answer.

Problem 4

(i). Consider a graph $G = (V, E)$. The line graph $L(G)$ of G is defined as follows: Each vertex in $L(G)$ corresponds to an edge in G, and two vertices are connected by an edge in $L(G)$ only if the corresponding edges in G are adjacent. Prove that if G has a Hamiltonian circuit, then $L(G)$ also has a Hamiltonian circuit.

(ii). Find an example of a graph G such that G does not have a Hamiltonian circuit but $L(G)$ does.

Problem 5

Show that if G is a connected graph with n vertices and minimum degree δ such that $n > 2\delta$ then G has a path of length at least 2δ.