

Computer Science 341

Discrete Mathematics

Problem Session 10

Mon, Dec 2, 2002

Problem 1

Suppose $G = (X \cup Y, E)$ is a bipartite graph where for all $x \in X$ the degree $d(x) \geq k$, and for all $y \in Y$ the degree $d(y) \leq 2k$. Show that G has a matching of size at least $\frac{|X|}{2}$.

Problem 2

A *random graph* on n vertices can be built as follows. For each pair of vertices (i, j) , toss a coin; if the outcome is heads, add an edge between i and j ; otherwise, don't. Show that a random graph with n vertices is almost surely connected when n is large.

Problem 3

You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability .8; with water it will die with probability .15. You are 90 percent certain that your neighbor will remember to water the plant.

- a. What is the probability that the plant will be alive when you return?
- b. If it is dead, what is the probability that your neighbor forgot to water it?

Problem 4

In the following problems, assume that any child is equally likely to be male or female and that the gender of any one child in a family is independent of the gender of any other children in that family.

- a. You visit the home of an acquaintance, who says, "I have two kids." A boy walks into the room. The acquaintance says, "That's my older child." What is the probability that the younger one is a boy?
- b. You go to a parent-teacher meeting. The principal is sitting in the first row. You've heard that the principal has two children. The teacher in charge asks everyone who has a son (meaning at least one) to raise a hand. The principal raises her hand. What is the probability that the principal has two sons?
- c. You live in a culture where, when children are introduced, male children are always introduced first, in descending order of age, and then female children, also in descending age order. You visit the home of an acquaintance, who says, "I have two kids, let me introduce them." He yells, "John, come here". What is the probability that the other child is a boy?

Problem 5

We toss a fair coin n times. What is the expected number of "runs"? Runs are consecutive tosses with the same result. For instance, the toss sequence HHHTTHTH has 5 runs.