COS 341 Discrete Mathematics

Administrative Information

- http://www.cs.princeton.edu/courses/archive/fall02/cs341/

- Professor: Moses Charikar moses@cs.princeton.edu
  305 CS building, 8-7477
  Office hours: Tue, 2:30-4:30

- Teaching Assistants:
  Adriana Karagiozova karagioz@cs.princeton.edu
  414 CS Building, 8-5388
  Office hours: Tue: 4:30-5:30

  Renato Werneck rwerneck@princeton.edu
  314 CS Building, 8-5135
  Office hours: Fri: 2:30-3:30

Discussion Sessions

- Discussion Session on Monday 5-6:00pm, 105 CS building
- Mailing list for class:
- Send mail to majordomo@cs.princeton.edu
- subscribe cs341   in the body

Proof Techniques: the pigeonhole principle

- n+1 pigeons in n holes
- There exists one hole with at least 2 pigeons

- Generalization: a pigeons in b holes
- There exists one hole with at least $\lceil$ **a/b** $\rceil$ pigeons

A nontrivial proof

Consider the numbers 1,2 … 1000. Show that amongst any 501 of them there exist two numbers such that one divides the other.

A nontrivial proof

Consider the numbers 1,2 … 1000. Show that amongst any 501 of them there exist two numbers such that one divides the other.

Write each number in the form

$2^k(2m+1), \quad k, m \geq 0$

Since m takes at most 500 distinct values,

the set contains two numbers of the form

$2^k(2m+1)$ and $2^{k'}(2m+1)$

Building blocks of logic

- Proposition: declarative sentence that is true or false (but not both).

  $x + y = z$
  $2 + 2 = 3$
  Today is Wednesday

- Basic building blocks of logic

- Usually denoted by lowecase letters: *p, q, r, s*
- Truth value of proposition denoted by T or F

Building New Propositions

Negation

p	$\neg$p
T	F
F	T

Truth table

Conjunction (AND)

p	q	p q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Disjunction (OR)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p implies q
if p, then q
q if p
q when p
q whenever p
q follows from p
p is sufficient for q
a sufficient condition for q is p
q is necessary for p
a necessary condition for p is q
p only if q

Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p if and only if q

p iff q

p is necessary and sufficient for q

if p then q and conversely

Translating from English

"You can access the internet from campus only if you are a computer science major or you are not a freshman"

a : You can access the internet from campus
c : You are a computer science major
f : You are a freshman

$a \rightarrow (c \vee \neg f)$

Precendence rules

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logical Equivalences

- A compound proposition that is always true is called a tautology.

- Propositions p and q are logically equivalent if they have the same truth values in all possible cases,
  i.e. if $p \leftrightarrow q$ is a tautology
- This is denoted by the notation $p \equiv q$

- Proved by verifying that truth tables agree or by using rules of logical equivalence

Equivalence by truth table

Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Logical Equivalences

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg (\neg p) \equiv p$	Double negation law

Logical Equivalences

Equivalence	Name
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws

Logical Equivalences

Equivalence	Name
$\neg (p \land q) \equiv \neg p \lor \neg q$ $\neg (p \lor q) \equiv \neg p \land \neg q$	De Morgan's laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv T$ $p \land \neg p \equiv F$	Negation laws

Applying equivalence laws: example

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

$(p \wedge q) \rightarrow (p \vee q)$

$\equiv \neg (p \wedge q) \vee (p \vee q)$	example
$\equiv (\neg p \vee \neg q) \vee (p \vee q)$	first De Morgan's Law
$\equiv (\neg p \vee p) \vee (\neg q \vee q)$	associative and commutative laws
$\equiv T \vee T$	
$\equiv T$	domination law

Other Equivalences

Implication

$$p \rightarrow q \equiv \neg p \vee q$$
$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$
$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$
$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$

Biconditionals

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
$$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$
$$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Quantifiers

- Universal quantifier $\forall$
  $\forall x \ P(x)$

- Existential quantifier $\exists$
  $\exists x \ P(x)$

Negation of quantifiers

$$\neg \forall x \ P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x \ Q(x) \equiv \forall x \neg Q(x)$$

Rules of inference

Justification of steps used to show conclusion follows logically from a set of hypothesis.

e.g. The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ gives the following rule of inference called modus ponens

$$\begin{array}{l} \quad p \\ \quad p \rightarrow q \\ \hline \therefore q \end{array}$$

Rule of inference	Name
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism