Homework 3: due Dec 13

Problem 1

1. Show that any 2-colorable graph can be 2-colored in polynomial time.

2. Show that any graph of maximum degree Δ can be colored with $\Delta + 1$ colors in polynomial time.

Problem 2 Show how to color a 3-colorable graph in $O(\sqrt{n})$ colors in polynomial time.

Problem 3 Let G be a graph with n vertices (no loops, mo multiple edges).

1. Prove that if G is k-colorable, then it is possible to assign a unit-length vector x_i of $\mathbf{R}^n$ to each vertex v_i, in such a way that $x_i^T x_j \leq -1/(k-1)$ for any two adjacent vertices v_i and v_j. (Hint: look for vectors with only k nonzero coordinates.)

2. Prove that if G has chromatic number $k+1$, such an assignment is not necessarily possible.