CS 487 – Midterm

Please turn in the midterm answers before 5:00 PM on Friday, October 26, 2001 to Ginny Hogan in Room 220. Be sure to put your name and "CS 487" on each page of your exam. You may keep the exam questions.

You are expected to follow the Princeton honor code. On your exam please write and sign the following statement: "I pledge my honor that I have not violated the honor code during this examination."

You may not discuss this exam with anyone except the instructor and TA. You may only use the following sources of information

- The Sipser textbook and its errata web pages.
- Notes from the lectures and precepts.
- The assignments and your solutions.
- The course web pages.

You may assume without proof any theorem proven in the above sources.

If you have any questions about the exam please contact the instructor or TA, preferably by email. We will respond to you quickly. Important clarifications or corrections will be emailed and posted to the course web page.

Be sure to answer all of the questions on both pages.

Justify all of your answers.

1. (10 points) According to "The **C** Programming Language,"

   > A floating point constant consists of an integer part, a decimal point, a fraction part, an `e` or `E`, and an optionally signed integer exponent. The integer and fraction parts both consist of a sequence of digits. Either the integer part or the fraction part (not both) may be missing; either the decimal point or the `e` and the exponent (not both) may be missing.

   Give a regular expression and an NFA that is equivalent to this definition.

2. Consider a 2-stack push-down automaton (2SPDA) which is a push-down automaton that has access to two distinct stacks.

   (a) (5 points) Give a formal definition of a 2SPDA and when it accepts.

   (b) (15 points) Show that there is a language L recognized by a 2SPDA but not context-free.

3. (10 points each) For each of the following languages classify them as either regular, context-free but not regular, or not context free.

   (a) $\{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } k \leq \min(i, j)\}$.

(b) $\{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } k \leq \max(i, j)\}$.

(c) $\{x\#x \mid x \text{ in } (0 \cup 1)^* \text{ and } 84 \leq |x| \leq 293\}$.

4. (20 points) Let $\mathbf{perm}(x)$ be the set of all permutations of the characters of a string x. For example,

$$\mathbf{perm}(0011) = \{0011, 0101, 0110, 1100, 1010, 1001\}.$$

For a language L we let

$$\mathbf{perm}(L) = \bigcup_{x \in L} \mathbf{perm}(x).$$

Do there exist any regular L such that $\mathbf{perm}(L)$ is not regular?

5. (20 points) Define $\mathbf{max}(L)$ as follows

$$\mathbf{max}(L) = \{x \mid x \text{ in } L \text{ and for all } y, |y| > 0, xy \text{ is not in } L\}.$$

Do there exist any regular L such that $\mathbf{max}(L)$ is not regular?