**CS 487 – Assignment 7**

1. Recall that $A \leq_m^p B$ if there is a polynomial-time computable $f$ such that $x$ is in $A$ if and only if $f(x)$ is in $B$. A language $L$ is NP-complete if $L$ is in NP and for all $A$ in NP, $A \leq_m^p L$.
   (a) Show that polynomial-time computable functions are closed under composition.
   (b) Show that if $A \leq_m^p B$ and $B \leq_m^p C$ then $A \leq_m^p B$.
   (c) Show that if $A$ is NP-complete, $A \leq_m^p B$ and $B$ is in NP then $B$ is NP-complete.
2. PARTITION is the problem where given a list of integers in binary, is it possible to partition the elements into two groups with the same sum. Show that PARTITION is NP-complete. Hint: Reduce from SUBSET SUM.
3. THREE COLORING consists of the set of graphs that are three colorable, i.e., there is a mapping C from vertices to {Red, Green, Blue} such that if $\{u, v\}$ is an edge then $C(u) \neq C(v)$. Show that THREE COLORING is NP-complete. Hint: Use the subgraphs given in problem 7.34 on page 275 of Sipser.
4. Show that if P = NP then there is a polynomial-time computable function $f$ such that given a satisfiable Boolean formula $\phi$, $f(\phi)$ is a satisfying assignment of $\phi$.
5. PRIMES consists of the set of numbers $m$ written in binary such that there are no $u$, $1 < u < m$ with $u$ dividing $m$. Show that PRIMES is in NP. You may use the following result from number theory without proof.

   Theorem: A number $p > 2$ is prime if and only if there is a number $r$, $1 < r < p$ such that $p$ divides $r^{p-1} - 1$ and $p$ does not divide $r^{\frac{p-1}{q}} - 1$ for all prime divisors $q$ of $p - 1$.