CS 487 – Assignment 5

A set A is co-recognizable if $\overline{A}$ is recognizable.

1. Recall the definition of 2SPDA from the midterm. Show that L is recognized by a 2SPDA if and only if L is recognizable.

2. A set A seperates sets B and C if $B \subseteq A$ and $C \subseteq \overline{A}$.

   Show that every pair of disjoint co-recognizable sets have a decidable seperator.

3. Show there exist two disjoint recognizable sets that have no decidable seperator.

4. Consider the set

$$\text{INF} = \{ \langle M \rangle \mid L(M) \text{ is infinite} \}.$$

   (a) Show that HALT $\leq$ INF.

   (b) Show that HALT $\leq \overline{\text{INF}}$.

   (c) Conclude that INF is neither recognizable or co-recognizable.

5. Let DIAG = $\{ \langle M \rangle \mid M$ on input $\langle M \rangle$ does not accept$\}$.

   (a) Suppose that for some Turing machine M, $L(M) \subseteq$ DIAG. Show that $\langle M \rangle$ is in DIAG - $L(M)$.

   (b) Using this result, show that there exists a computable function such that given the description of a consistent theory produces a true statement that is not provable in that theory.

   (c) You have just proven that a statement is true and has no proof. Why is this not a logical contradiction?