CS 487 – Assignment 4

1. Consider the restricted sets of context free grammars where the production rules only take the form

   - $A \to aB$
   - $A \to \epsilon$

   for A and B in V and a in Σ.

   Prove that a language is regular if and only if it has such a grammar.

2. Consider a modification of a push-down automaton we will call an AESPDA (Accept by Empty Stack PDA) as follows:

   - No specified accept states.
   - Computation starts with special stack symbol "$" on stack.
   - Accepts when input has been read and stack is empty.

   (a) Give a formal definition of an AESPDA, including a formal definition of when it accepts.

   (b) Show that AESPDAs accept exactly the context-free languages.

   (c) Show that every context-free language has an AESPDA with only one state.

3. Which of the following languages are context-free? Prove your answers.

   (a) $L_1 = \{0^n \mid n \text{ is prime}\}$.

   (b) $L_2 = \{x \in (0 \cup 1)^* \mid x \text{ has the same number of 0's and 1's}\}$.

   (c) $L_3 = \{x \in (0 \cup 1 \cup 2)^* \mid x \text{ has the same number of 0's, 1's and 2's}\}$.

   (d) $L_4 = \{0^i 1^j \mid i \neq j\}$.

   (e) $L_5 = \{0^i 1^j \mid i \neq j \text{ and } i \neq 2j\}$.

   (f) $L_6 = \{xy \mid x, y \in (0 \cup 1)^*, |x| = |y| \text{ and } x \neq y\}$.

4. Give a proof or counterexample that context-free grammars are closed under

   (a) Union.

   (b) Intersection.

   (c) *-operation.

   (d) Complement.

   (e) Concatenation.

5. Give a formal description of a Turing machine that accepts the language

$$\{w \mid w \text{ has the same number of 0's and 1's}\}.$$