CS 487 – Final

Please turn in the answers before 4:00 PM on Friday, January 18, 2002, to Ginny Hogan in Room 220. Be sure to put your name and "CS 487" on each page of your exam. You may keep the exam questions.

You are expected to follow the Princeton honor code. On your exam please write and sign the following statement: "I pledge my honor that I have not violated the honor code during this examination."

You may not discuss this exam with anyone except the instructor and TA. You may only use the following sources of information

- The Sipser textbook and its errata web pages.
- Notes from the lectures and precepts.
- The assignments and your solutions.
- The course web pages.

You may assume without proof any theorem proven in the above sources.

If you have any questions about the exam please contact the TA or instructor preferably by email. Please look over the questions early because the TA and instructor may not be available on Friday. Important clarifications or corrections will be emailed and posted to the course web page.

Justify all of your answers.

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A perfect score is 80 points.

1. (5 points each) Which of the following languages are decidable?

   (a) $\{\langle M\rangle \mid$ M is a Turing machine and $L(M)$ is a context-free language$\}$.

   (b) $\{\langle M\rangle \mid$ M is a one-tape Turing machine and M only uses at most the first fifty tape cells on every input$\}$.

   (c) $\{\langle M\rangle \mid$ M is a Turing machine and $L(M) = \overline{L(M)}\}$.

   (d) $\{\langle M\rangle \mid$ M is a Turing machine and it enters state q_7 at least twice on input $\epsilon\}$.

2. (5 points each) Let Σ be a finite alphabet and let $a \in \Sigma$. We define the *a-projection* of $w \in \Sigma^*$ as the string w_a obtained from w by deleting all occurrences of the letter a. The a-projection of a language $L \subseteq \Sigma^*$ is the language $L_a = \{w_a : w \in L\}$.

   (a) Prove that the a-projection of a regular language is regular.

   (b) Prove that the a-projection of a context-free language is context-free.

   (c) Prove that the a-projection of a recognizable language is recognizable.

   (d) Prove that the a-projection of a decidable language is not necessarily decidable. (Construct a counterexample and prove it.)

3. (20 points) A matrix is *Quiverth* if it contains only entries from $\{-1, 0, 1\}$ and some subset of its rows sum to a vector with only negative entries. More formally, let A be an n by m matrix with indices labelled by a_{ij}. Then A is Quiverth if

   (i) for each i, $1 \le i \le n$ and j, $1 \le j \le m$, $a_{ij} \in \{-1, 0, 1\}$, and

   (ii) there is an $S \subseteq \{1, \ldots, n\}$ such that for all j, $1 \le j \le m$,

$$\sum_{i \in S} a_{ij} < 0.$$

   Show either (a) or (b):

   (a) The set of Quiverth matrices is in P.

   (b) The set of Quiverth matrices is NP-complete.

   Extra credit for showing both.

4. (20 points) A linear bounded automaton is a nondeterministic Turing machine that has a single tape and cannot move its head past the end of the input. It can replace the input bits with other tape symbols.

   Show that linear bounded automata accept exactly the languages in NSPACE(n). What is the relationship between the sets accepted by linear bounded automata and PSPACE?