Image Processing

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Overview
• Image representation
  ◦ What is an image?
• Halftoning and dithering
  ◦ Trade spatial resolution for intensity resolution
  ◦ Reduce visual artifacts due to quantization
• Sampling and reconstruction
  ◦ Key steps in image processing
  ◦ Avoid visual artifacts due to aliasing

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What is an Image?
• An image is a 2D rectilinear array of pixels

3

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4

Image Acquisition
• Pixels are samples from continuous function
  ◦ Photoreceptors in eye
  ◦ CCD cells in digital camera
  ◦ Rays in virtual camera

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A pixel is a sample, not a little square!

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Image Display
- Re-create continuous function from samples
  - Example: cathode ray tube

Image is reconstructed by displaying pixels with finite area (Gaussian)

Image Resolution
- Intensity resolution
  - Each pixel has only "Depth" bits for colors/intensities
- Spatial resolution
  - Image has only "Width" x "Height" pixels
- Temporal resolution
  - Monitor refreshes images at only "Rate" Hz

<table>
<thead>
<tr>
<th>Typical Resolutions</th>
<th>Width x Height</th>
<th>Depth</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTSC</td>
<td>640 x 480</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Workstation</td>
<td>1280 x 1024</td>
<td>24</td>
<td>75</td>
</tr>
<tr>
<td>Film</td>
<td>3000 x 2000</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Laser Printer</td>
<td>6600 x 5100</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Sources of Error
- Intensity quantization
  - Not enough intensity resolution
- Spatial aliasing
  - Not enough spatial resolution
- Temporal aliasing
  - Not enough temporal resolution

\[ E^2 = \sum_{(x,y)} (I(x, y) - P(x, y))^2 \]

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  - What is an image?
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      - Reduce visual artifacts due to quantization
    - Sampling and reconstruction
      - Reduce visual artifacts due to aliasing

Quantization
- Artifacts due to limited intensity resolution
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range

Uniform Quantization
\[ P(x, y) = \text{trunc}(I(x, y) + 0.5) \]
Uniform Quantization

- Images with decreasing bits per pixel:

  8 bits  4 bits  2 bits  1 bit

Notice contouring

Reducing Effects of Quantization

- Halftoning
  - Classical halftoning

- Dithering
  - Random dither
  - Ordered dither
  - Error diffusion dither

Classical Halftoning

- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image

Classical Halftoning

Newspaper Image

From New York Times, 9/21/99

Halftone patterns

- Use cluster of pixels to represent intensity
  - Trade spatial resolution for intensity resolution

Dithering

- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

Figure 14.37 from H&B
Random Dither
- Randomize quantization errors
  - Errors appear as noise

\[ P(x, y) = \text{trunc}(I(x, y) + \text{noise}(x,y) + 0.5) \]

Ordered Dither
- Pseudo-random quantization errors
  - Matrix stores pattern of thresholds

\[ i = x \mod n \]
\[ j = y \mod n \]
\[ e = I(x,y) - \text{trunc}(I(x,y)) \]
\[ \text{if} (e > D(i,j)) \]
\[ P(x,y) = \text{ceil}(I(x,y)) \]
\[ \text{else} \]
\[ P(x,y) = \text{floor}(I(x,y)) \]

Ordered Dither
- Bayer's ordered dither matrices

\[ D_x = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \]
\[ D_y = \begin{bmatrix} 4D_x & 4D_z \\ 4D_y & 4D_z \end{bmatrix} \]

\[ D_z = \begin{bmatrix} 15 & 7 & 13 & 5 \\ 3 & 11 & 1 & 9 \\ 12 & 4 & 14 & 6 \\ 0 & 8 & 2 & 10 \end{bmatrix} \]

Error Diffusion Dither
- Spread quantization error over neighbor pixels
  - Error dispersed to pixels right and below

\[ \alpha + \beta + \gamma + \delta = 1.0 \]

Figure 14.42 from H&B
### Error Diffusion Dither

- Original (8 bits)
- Random Dither (1 bit)
- Ordered Dither (1 bit)
- Floyd-Steinberg Dither (1 bit)

### Overview

- Image representation
  - What is an image?
- Halftoning and dithering
  - Reduce visual artifacts due to quantization
  - Reduce visual artifacts due to aliasing

### Sampling and Reconstruction

![Sampling Diagram]

- Sampling
- Reconstruction

### Aliasing

- In general:
  - Artifacts due to under-sampling or poor reconstruction
- Specifically, in graphics:
  - Spatial aliasing
  - Temporal aliasing

### Spatial Aliasing

- Artifacts due to limited spatial resolution
Spatial Aliasing
- Artifacts due to limited spatial resolution

```
Jaggies
```

Temporal Aliasing
- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering

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Antialiasing
- Sample at higher rate
  - Not always possible
  - Doesn’t always solve problem

- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)
  - Trades aliasing for blurring

Must consider sampling theory!
Sampling Theory
- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?

Spectral Analysis
- Spatial domain:
  - Function: \( f(x) \)
  - Filtering: convolution
- Frequency domain:
  - Function: \( F(u) \)
  - Filtering: multiplication

Any signal can be written as a sum of periodic functions.

Fourier Transform
- Fourier transform:
  \[
  F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx
  \]
- Inverse Fourier transform:
  \[
  f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du
  \]

Sampling Theorem
- A signal can be reconstructed from its samples, if the original signal has no frequencies above 1/2 the sampling frequency - Shannon
- The minimum sampling rate for bandlimited function is called “Nyquist rate”

Convolution
- Convolution of two functions (= filtering):
  \[
  g(x) = f(x) \ast h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x-\lambda) d\lambda
  \]
- Convolution theorem
  - Convolution in frequency domain is same as multiplication in spatial domain, and vice-versa

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.
Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

- Consider reducing the image resolution

Antialiasing in Image Processing

- General Strategy
  - Pre-filter transformed image via convolution with low-pass filter to form bandlimited signal
- Rationale
  - Prefer blurring over aliasing

Image Processing

- Image processing is a resampling problem

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### Ideal Low-Pass Filter

- Frequency domain
  - ![Frequency Domain Plot](image1.png)
  
- Spatial domain
  - ![Spatial Domain Plot](image2.png)
  
*Figure 4.5 Wolberg*

### Practical Image Processing

- Finite low-pass filters
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter

*Real world → Sample → Discrete samples (pixels) → Reconstruct → Reconstructed function → Transform → Transformed function → Filter → Bandlimited function → Sample → Discrete samples (pixels) → Reconstruct → Display*

*Figure 2.4 Wolberg*

### Triangle Filter

- Convolution with triangle filter

*Input → Output*

*Figure 2.4 Wolberg*

### Gaussian Filter

- Convolution with Gaussian filter

*Input → Output*

*Figure 2.4 Wolberg*

### Image Processing

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*Figure 2.4 Wolberg*

### Adjusting Brightness

- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 255)

*Original → Brighter*

*Figure 2.4 Wolberg*
Adjusting Contrast

- Compute mean luminance \( \bar{l} \) for all pixels
  - \( \text{luminance} = 0.30 \times r + 0.59 \times g + 0.11 \times b \)
- Scale deviation from \( \bar{l} \) for each pixel component
  - Must clamp to range (e.g., 0 to 255)

![Original](Image) ![More Contrast](Image)

Image Processing

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Adjust Blurriness

- Convolve with a filter whose entries sum to one
  - Each pixel becomes a weighted average of its neighbors

![Original](Image) ![Blur](Image)

Filter: \[
\begin{bmatrix}
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16}
\end{bmatrix}
\]

Edge Detection

- Convolve with a filter that finds differences between neighbor pixels

![Original](Image) ![Detect edges](Image)

Filter: \[
\begin{bmatrix}
-1 & -1 & -1 \\
-1 & 8 & -1 \\
-1 & -1 & -1
\end{bmatrix}
\]

Image Processing

- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Pixel operations
  - Add random noise
  - Add luminance
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Scaling

- Resample with triangle or Gaussian filter

![Original](Image) ![1/4X resolution](Image) ![4X resolution](Image)

More on this next lecture!
Image Processing

- Image processing is a resampling problem
  - Avoid aliasing
  - Use filtering

Summary

- Image representation
  - A pixel is a sample, not a little square
  - Images have limited resolution

- Halftoning and dithering
  - Reduce visual artifacts due to quantization
  - Distribute errors among pixels
    - Exploit spatial integration in our eye

- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing
  - Filter to avoid undersampling
    - Blurring is better than aliasing