COS 341 Homework 1 Solutions

Section 1.8

Problem 3. It's not possible. One square will always go unvisited. In order to visit each square the prisoner must make 63 moves. Each move goes from white to black or black to white. Because 63 is odd, he must end on a color different than that on which he started. But both the ending and starting cells are the same color.

Problem 13.

47	58	69	80	1	12	23	34	45
57	68	79	9	11	22	33	44	46
67	78	8	10	21	32	43	54	56
77	7	18	20	31	42	53	55	66
6	17	19	30	41	52	63	65	76
16	27	29	40	51	62	64	75	5
26	28	39	50	61	72	74	4	75
36	38	49	60	71	73	3	14	25
37	48	59	70	81	2	13	24	35

Problem 16. Label the squares as follows:

2	3	w	x
4	a	b	c
y	d	e	f
z	g	h	i

By the formula given in section 1.3, the magic sum of a magic square of order 4 is 34. Hence, we have

$$\begin{aligned} 2+3+w+x &= 34 \Rightarrow w+x=29 \\ 2+4+y+z &= 34 \Rightarrow y+z=28 \end{aligned}$$

The only possibilities for $\{w, x\}$ are $\{13, 16\}$ and $\{14, 15\}$. The only possibilities for $\{y, z\}$ are $\{12, 16\}$ and $\{13, 15\}$. But $w \neq x \neq y \neq z$, so $\{w, x\} = \{14, 15\}$ and $\{y, z\} = \{12, 16\}$.

The remaining numbers are 1,5,6,7,8,9,10,11,13. We know

$$\begin{aligned} a+b+c &= 30 \\ a+d+g &= 31 \\ a+e+i &= 32 \end{aligned}$$

The possible ways to sum to 30 from the remaining numbers are

$$\begin{aligned} 13+11+6 &= 30 \\ 13+10+7 &= 30 \\ 13+9+8 &= 30 \\ 11+10+9 &= 30 \end{aligned}$$

The possible ways to sum to 31 from the remaining numbers are

$$\begin{aligned} 13+11+7 &= 31 \\ 13+10+8 &= 31 \end{aligned}$$

The possible ways to sum to 32 from the remaining numbers are

$$\begin{aligned} 13+11+8 &= 32 \\ 13+10+9 &= 32 \end{aligned}$$

We need to pick one equation from each group above so that they share exactly one number. Of the four ways to choose one equation from the second group and one equation from the third group, there is only one way that uses only one number twice:

$$\begin{aligned} 13+10+9 &= 30 \text{ and} \\ 13+11+7 &= 31 \end{aligned}$$

But all equations in the first group use at least two of {7,9,10,11,13}. So there is no way to choose three appropriate equations.

Problem 31. The game is not balanced. We have

$$\begin{array}{rcl} \texttt{10} & == & \texttt{001010} \\ \texttt{20} & == & \texttt{010100} \\ \texttt{30} & == & \texttt{011110} \\ \texttt{40} & == & \texttt{101000} \\ \texttt{50} & == & \texttt{110010} \\ \hline \text{sum} & & \texttt{2}\mathbf{3323}\texttt{0} \end{array}$$

We need to make the bold numbers even.

The 30 pile has a 1 in each of these columns, and so we can try to make that pile be $000100 = 4$. Thus, player 1 should take 26 coins from the pile with 30 coins in it.

Problem 33. Let $N = x_k x_{k-1} ... x_1 x_0$ be the binary representation of the number of coins in a pile P such that $2^j <= N < 2^{j+1}$.

Construct N' from N by complementing each bit that is unbalanced.

$N' < N$ since the jth bit of N is 1 but will be complemented to 0 and since j is the largest bit that is unbalanced, no larger bits will be complemented.

If player I removes $C = N - N'$ coins from pile P, the game will be balanced because after removing C coins, pile P will contain N' coins and each of the columns will be balanced because by the construction of N', each unbalanced column will contain one more or one fewer coins.